

12-2 Undefined and Simplifying Rational Expressions

expression which can be written in the form $\frac{\text{polynomial}}{\text{polynomial}}$ is a rational expression.

Examples of rational expressions are below.

$$\frac{1}{x}$$

$$\frac{x+2}{2x-8}$$

$$\frac{x^2-5}{x^2-10x+25}$$

Before we begin simplifying these rational expressions, we must *determine which value(s) will make each fraction **undefined***. Undefined fractions have a denominator of 0. To determine which value(s) will make the fraction undefined, you can set the denominator equal to 0 and solve for the variable.

$$\frac{4}{0}$$

$$\frac{1}{x}$$

$$x = 0$$

$$\frac{x+2}{2x-8}$$

$$\begin{aligned} 2x-8 &= 0 \\ +8 & \quad +8 \\ \hline 2x &= 8 \\ \frac{2x}{2} &= \frac{8}{2} \\ x &= 4 \end{aligned}$$

$$\frac{x^2-5}{x^2-10x+25}$$

1, 25

$$\begin{aligned} x^2-10x+25 &= 0 \\ (x-5)(x-5) &= 0 \\ x-5 &= 0 & x-5 &= 0 \\ x &= 5 & x &= 5 \end{aligned}$$

A rational expression is in simplest form if the numerator and denominator have no common factors except 1. Factor both the numerator and denominator (if necessary) by **FIRST factoring out the GCF**.

$$1. \frac{6x+12}{x+2} = \frac{6(x+2)}{(x+2)} = \frac{6}{1} = 6$$

$$2. \frac{2x}{4x^2-6x} = \frac{2x}{2x(2x-3)} = \frac{1}{(2x-3)}$$

$$\frac{4m-2}{2m-1} = \frac{2(2m-1)}{(2m-1)} = \frac{2}{1} = 2$$

You may also need to factor a quadratic expression to simplify a rational expression.

$$4. \frac{2x-12}{x^2-3x-18} = \frac{2(x-6)}{(x+3)(x-6)} = \frac{2}{(x+3)}$$

$$5. \frac{c^2-c-6}{c^2+5c+6} = \frac{(c+2)(c-3)}{(c+2)(c+3)} = \frac{(c-3)}{(c+3)}$$

$$6. \frac{x^2+9x-10}{x^3-x^2} = \frac{(x+10)(x-1)}{x^2(x-1)} = \frac{(x+10)}{x^2}$$

So what if you factor the expression and opposite factors appear such as $\frac{(3-x)}{(x-3)}$.

$$\frac{(x-3)}{(x-3)} = 1$$

$$\frac{(x+3)}{(x+3)} = 1$$

$$\frac{(3+x)}{(x+3)} = 1$$

$$\frac{*(3-x)}{(x-3)} = -1$$

$$8. \frac{5x-15}{9-x^2} = \frac{5(x-3)}{(3+x)(3-x)} = \frac{-5}{(3+x)}$$

$$9. \frac{2c^2-2}{3-3c^2} = \frac{2(c^2-1)}{3(1-c^2)} = \frac{2(c+1)(c-1)}{3(1+c)(1-c)} = \frac{-2}{3}$$

For the problems below, simplify the expression in Column A and match it with the equivalent expression in Column B

Column A	Show Work	Column B
$\frac{3x-9}{3x}$		$\frac{x-1}{y-1}$
$\frac{6x-9}{4x-6}$		$\frac{x+2}{x+4}$
$\frac{x^2-x}{xy-x}$		$\frac{3}{2}$
$\frac{x+7}{x^2-49}$		$\frac{x+3}{5}$
$\frac{x^2+7x+12}{5x+20}$		$\frac{x-3}{x}$
$\frac{x^2-x-6}{x^2+x-12}$		$\frac{2(x+2)}{8+x}$
$\frac{2x^2-12x-32}{64-x^2}$		x
$\frac{x^5-x^4}{x^4-x^3}$		$\frac{1}{x-7}$