

Definition: The ratio of two numbers a and b , where b is not zero is the number $\frac{a}{b}$. (Also written $a : b$)

Definition: A **proportion** is an equation that states that two ratios are equal $\frac{a}{b} = \frac{c}{d}$ or $a : b = c : d$

The four numbers, a , b , c , and d are *terms* in the proportion. The first and fourth terms, a and d are the extremes of the proportion. The second and third terms, b and c , are the means of the proportion.

$$\begin{array}{ccc}
 & \text{Means} & \\
 & \downarrow & \\
 a : b = c : d & & \\
 \uparrow & & \uparrow \\
 & \text{Extremes} &
 \end{array}
 \qquad
 \begin{array}{ccc}
 \frac{a}{b} & \leftarrow \text{Extremes} & \rightarrow \frac{c}{d} \\
 & \text{Means} & \\
 & \rightarrow & \\
 & \text{Means} & \\
 & \leftarrow & \\
 & \text{Extremes} &
 \end{array}$$

$$ad = bc$$

Cross-Product Property: The product of the extremes is equal to the product of the means.

In a proportion, the means may be interchanged and the extremes may be interchanged. The

proportion $\frac{4}{8} = \frac{11}{22}$ can be rewritten as $\frac{4}{11} = \frac{8}{22}$ or $\frac{22}{8} = \frac{11}{4}$.

Mean Proportional (Geometric Mean)

If the two means of a proportion are equal, either mean is called the *mean proportional* or *geometric mean* between the first and fourth terms of the proportion.

In the proportion $\frac{2}{8} = \frac{8}{32}$, the number 8 is the geometric mean between 2 and 32.

Model Problems:

1. Find the mean proportional between the lengths 4 and 16.

$$\frac{4}{x} = \frac{x}{16}$$

$$x^2 = 64$$

Answer: $x = 8$

Check: $\frac{4}{8} = \frac{8}{16}$

$$\frac{1}{2} = \frac{1}{2}$$

2. Find the geometric mean between the lengths 8 and 12.

$$\frac{8}{x} = \frac{x}{12}$$

$$x^2 = 96$$

$$x = \sqrt{96}$$

$$x = \sqrt{16} \cdot \sqrt{6}$$

Answer: $x = 4\sqrt{6}$

Mixed Practice

1. Find the mean proportional between the lengths 6 and 16.

2. Find the geometric mean between the lengths 19 and 76.

3. You want to make a scale drawing of your bedroom to help you arrange your furniture. You decide on a scale of 3 in. = 2 ft. Your bedroom is a 12 ft.-by-15 ft. rectangle. What should be its dimensions in your scale drawing.

4. $\triangle ABC \sim \triangle DEF$. If $AB = 6$, $DE = 8$, and $DF = 12$, find AC .

5. The sides of a triangle measure 2, 3, and 4. If the smallest side of a similar triangle measures 8, then find the measure of the largest side.

6. $\triangle ABC \sim \triangle DEF$. If $AB = 3$, $BC = 12$, $DE = x + 2$, and $EF = 18$, find the value of x .

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Mixed Practice

1. Find the mean proportional between the lengths 6 and 16.

$$\frac{6}{x} = \frac{x}{16}$$

$$x^2 = 96$$

$$x = \sqrt{96}$$

$$x = \sqrt{16 \cdot 6}$$

$$x = 4\sqrt{6}$$

2. Find the geometric mean between the lengths 19 and 76.

$$\frac{19}{x} = \frac{x}{76}$$

$$x^2 = 1444$$

$$x = 38$$

3. You want to make a scale drawing of your bedroom to help you arrange your furniture. You decide on a scale of 3 in. = 2 ft. Your bedroom is a 12 ft.-by-15 ft. rectangle. What should be its dimensions in your scale drawing.

$$\frac{3 \text{ in}}{2 \text{ ft}} = \frac{x}{12 \text{ ft}}$$

$$2x = 36$$

$$x = 18$$

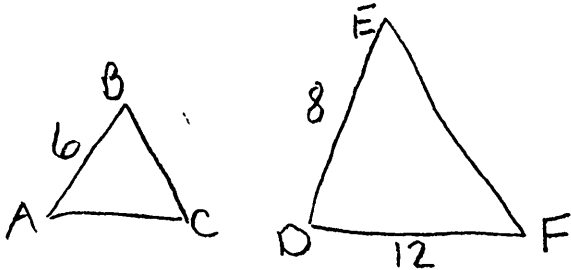
$$\frac{3 \text{ in}}{2 \text{ ft}} = \frac{x}{15 \text{ ft}}$$

$$2x = 45$$

$$x = 22.5$$

$$18 \text{ in} - \text{by} - 22.5 \text{ in}$$

4. $\triangle ABC \sim \triangle DEF$. If $AB = 6$, $DE = 8$, and $DF = 12$, find AC .



$$\frac{6}{8} = \frac{x}{12}$$

$$8x = 72$$

$$x = 9$$

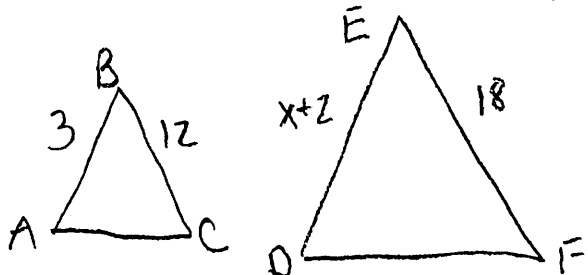
5. The sides of a triangle measure 2, 3, and 4. If the smallest side of a similar triangle measures 8, then find the measure of the largest side.

$$\frac{2}{8} = \frac{4}{x}$$

$$2x = 32$$

$$x = 16$$

6. $\triangle ABC \sim \triangle DEF$. If $AB = 3$, $BC = 12$, $DE = x + 2$, and $EF = 18$, find the value of x .



$$\frac{3}{x+2} = \frac{12}{18}$$

$$12(x+2) = 54$$

$$12x + 24 = 54$$

$$12x = 30$$

$$x = 2.5$$