7-4 Similarity in Right Triangles

∠C is a right angle

$\overline{CD}$ is an altitude to $\overline{AB}$

1 (The whole triangle)
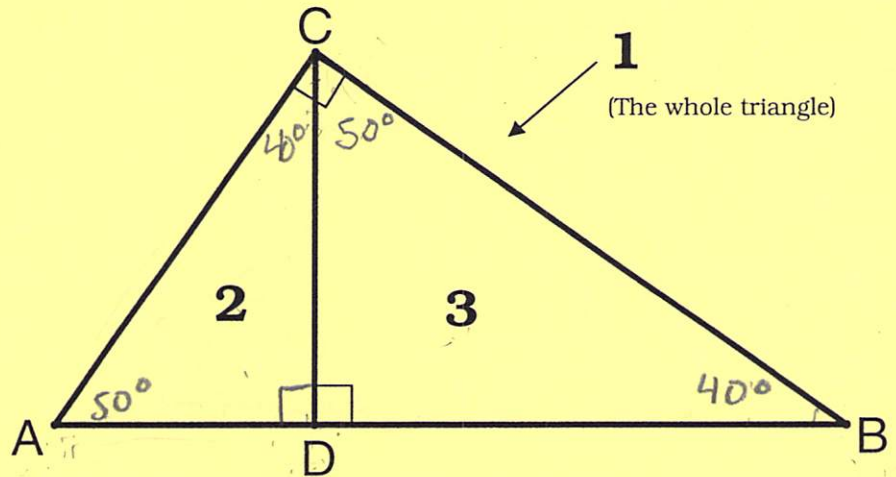
If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles that are formed are each similar to the large triangle as well as similar to each other. Let's see how this is possible...

Write the similarity statement and raw the triangles in corresponding order.

Δ1 ~ Δ2

△ _ABC_ ~ △ _ACD_

Δ1 ~ Δ3

△ _ABC_ ~ △ _CBD_

Δ2 ~ Δ3

△ _ACD_ ~ △ _CBD_

Refer to the diagram above to complete each proportion.

$\underset{\Delta 2}{\dfrac{AD}{CD}} = \underset{\Delta 3}{\dfrac{CD}{BD}}$ | $\underset{\Delta 1}{\dfrac{AB}{AC}} = \underset{\Delta 2}{\dfrac{AC}{AD}}$ | $\underset{\Delta 1}{\dfrac{AB}{BC}} = \underset{\Delta 3}{\dfrac{BC}{BD}}$ | $\underset{\Delta 2}{\dfrac{AD}{AC}} = \underset{\Delta 1}{\dfrac{AC}{AB}}$ | $\dfrac{BD}{BC} = \dfrac{BC}{AB}$

$\dfrac{SL}{LL} = \dfrac{SL}{LL}$ $\dfrac{Hyp}{SL} = \dfrac{Hyp}{SL}$

Solve for x.

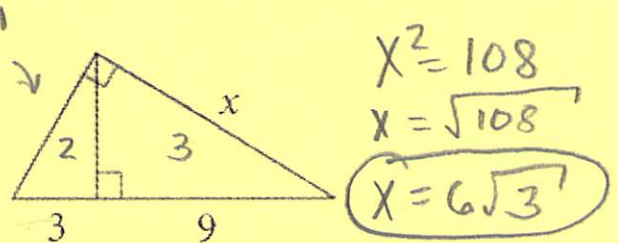1.

2.

$x^2 = 108$

$x = \sqrt{108}$

$\boxed{x = 6\sqrt{3}}$

$\underset{\Delta 2}{\dfrac{x}{6}} = \underset{\Delta 3}{\dfrac{6}{4}}$ $\boxed{x = 9}$

$\underset{\Delta 3}{\dfrac{9}{x}} = \underset{\Delta 1}{\dfrac{x}{12}}$

3.
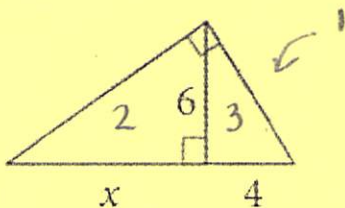
$x+3$
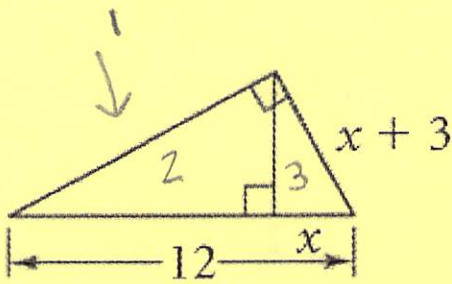
$$x^2 + 6x + 9 = 12x$$
$$-12x \qquad\qquad -12x$$

$$x^2 - 6x + 9 = 0$$

$$(x-3)(x-3) = 0$$

$$\boxed{x = 3}$$

$\triangle 1$ $\qquad$ $\triangle 3$

$$\frac{x}{x+3} = \frac{x+3}{12}$$

4. $\triangle ABC$ is a right triangle. $\angle C$ is a right angle and altitude $\overline{CD}$ is drawn to $\overline{AB}$. If $\overline{AB} = 6$ and $\overline{DB} = 24$

a. Find $\overline{CD}$ b. Find $\overline{AC}$

$\triangle 2$ $\qquad$ $\triangle 3$
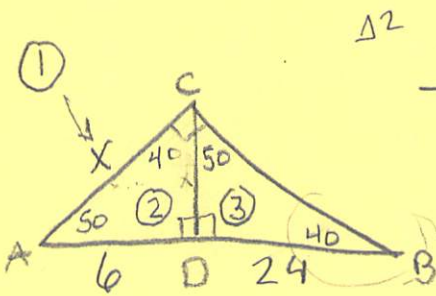
$$\frac{6}{x} = \frac{x}{24}$$

$$x^2 = 144$$
$$x^2 - 144 = 0$$
$$\boxed{x = 12}$$

$\triangle 1$ $\qquad$ $\triangle 2$

$$\frac{30}{x} = \frac{x}{6}$$

$$x^2 = 180$$

$$\boxed{x = 6\sqrt{5}}$$

5. $\triangle ABC$ is a right triangle. $\angle C$ is a right angle and altitude $\overline{CD}$ is drawn to $\overline{AB}$. If $\overline{CD} = 12$ and $\overline{AD}$ exceeds $\overline{DB}$ by 7.

a. Find $\overline{DB}$ b. Find $\overline{AD}$

$\triangle 2$ $\qquad$ $\triangle 3$

$$\frac{x+7}{12} = \frac{12}{x}$$

$$\boxed{\overline{DB} = 9}$$

$$x^2 + 7x = 144$$

$$x^2 + 7x - 144 = 0$$

$$(x-9)(x+16) = 0$$
$$x - 9 = 0 \quad\bigg|\quad x + 16 = 0$$
$$\boxed{x = 9} \quad\bigg|\quad x = -16$$

$$\overline{AD} = 9 + 7$$

$$\boxed{\overline{AD} = 16}$$