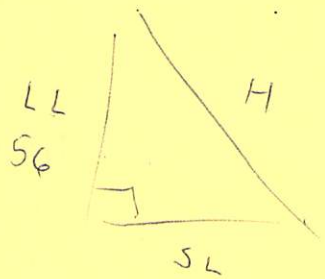
The Pythagorean Theorem and its Converse

Pythagorean Triples

A *Pythagorean triple* is a set of positive integers that when substituted for a, b, and c in the equation, $a^2 + b^2 = c^2$, make the equation true. To form a Pythagorean triple, choose any two positive integers U and V, $U > V$.

$$\text{Let } a = U^2 - V^2$$
$$b = 2UV$$
$$c = U^2 + V^2$$



For each given value of U and V, fill in the appropriate column in the table below.

		A	B	C		
U	**V**	$U^2 - V^2$	$2UV$	$U^2 + V^2$	$A^2 + B^2$	C^2
2	1	3	4	5	25	25
4	1	15	8	17	289	289
6	1	35	14	49	1421	1421
8	1	63	16	65	4225	4225
3	2	5	12	13	169	169
5	2	21	20	29	841	841
7	2	45	28	53	2809	2809
9	2	77	36	85	7225	7225
4	3	7	24	25	625	625
8	3	55	48	73	5329	5329

Some Challenges:

a. Consider the numbers 51, 140, 149 which form a Pythagorean Triple. What values of U and V give this triple?

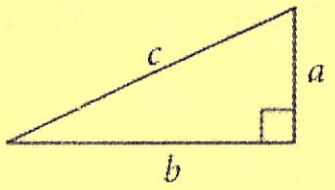
10 , 7

Write a Pythagorean Triple in which the middle number is 56. 33, 56, 65

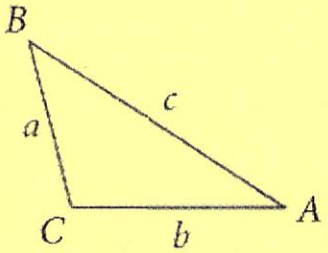
c. Write a Pythagorean Triple in which the smallest number is 69. 69, 260, 269

Examining the other uses of Pythagoreans Theorem

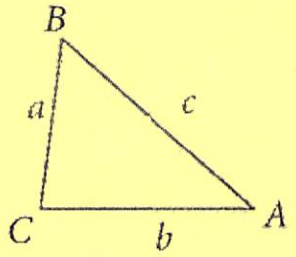
If $a^2 + b^2 = c^2$ then the triangle is a right triangle.



If $a^2 + b^2 < c^2$ then the triangle is an obtuse triangle.



If $a^2 + b^2 > c^2$ then the triangle is an acute triangle.



The lengths of the sides of a triangle are given. Classify the triangle as acute, right, or obtuse.

1. 4, 5, 6 ACUTE

4^2+5^2 6^2

$16+25$ 36

$41 > 36$

2. 0.3, 0.4, 0.6 OBTUSE

$.3^2+.4^2$ $.6^2$

$.09+.16$ $.36$

$.25 < .36$

3. 11, 12, 15 ACUTE

11^2+12^2 15^2

$121+144$ 225

$265 > 225$

4. $\sqrt{3}$, 2, 3 OBTUSE

$(\sqrt{3})^2+2^2$ 3^2

$3+4$ 9

$7 < 9$

5. 30, 40, 50 RIGHT

30^2+40^2 50^2

$900+1600$ 2500

$2500 = 2500$ ✓

6. $\sqrt{11}$, $\sqrt{7}$, 4 ACUTE

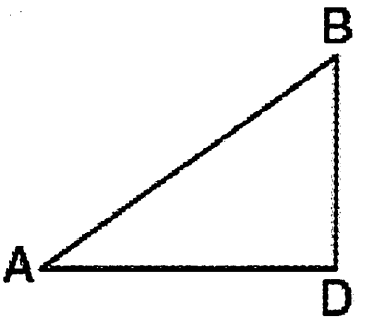
$(\sqrt{11})^2+(\sqrt{7})^2$ 4^2

$11+7$ 16

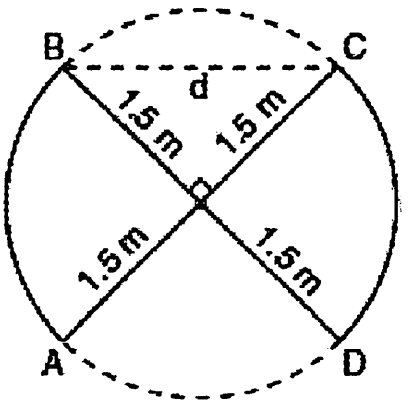
$18 > 16$

Pythagorean Theorem Practice Name _____

1. In the diagram below of ΔADB, $m\angle BDA = 90$, $AD = 5\sqrt{2}$, and $AB = 2\sqrt{15}$. What is the length of BD?



2. An overhead view of a revolving door is shown in the accompanying diagram. Each panel is 1.5 meters wide. What is the approximate width of d, the opening from B to C to the nearest hundredth of a meter?



3. Which set of numbers does *not* represent the sides of a right triangle?

(1) { 6, 8, 10 } (2) { 8, 15, 17 } (3) { 8, 24, 25 } (4) { 15, 36, 39 }

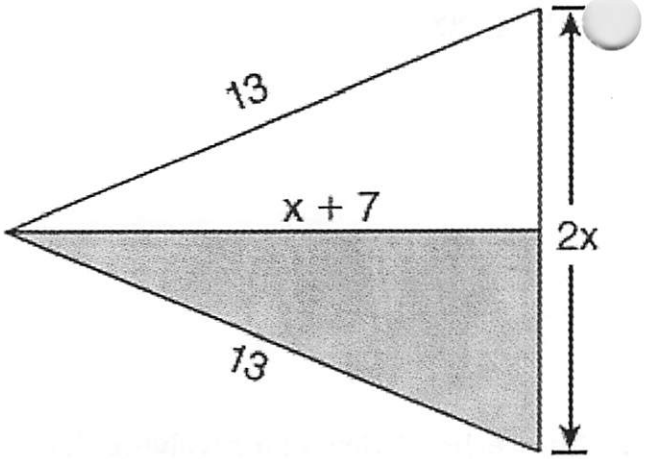
4. The set of integers {3,4,5} is a Pythagorean triple. Another such set is

(1) { 6, 7, 8 } (2) { 6, 8, 12 } (3) { 6, 12, 13 } (4) { 8, 15, 17 }

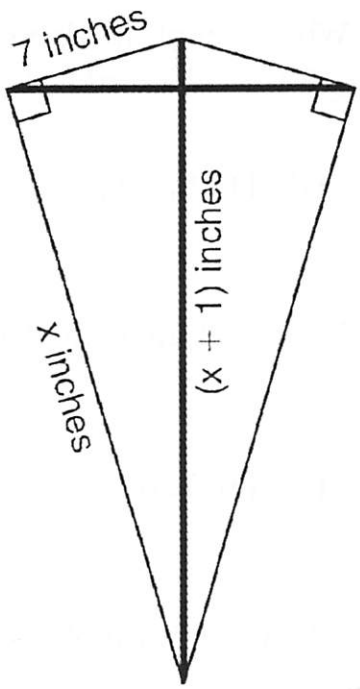
5. Which set of numbers could be the lengths of the sides of a right triangle?

(1) { 10, 24, 26 } (2) { 12, 16, 30 } (3) { 3, 4, 6 } (4) { 4, 7, 8 }

6. The diagram below shows a pennant in the shape of an isosceles triangle. The equal sides each measure 13, the altitude is $x + 7$, and the base is $2x$. What is the length of the base?



7. As shown in the diagram below, a kite needs a vertical and a horizontal support bar attached at opposite corners. The upper edges of the kite are 7 inches, the side edges are x inches, and the vertical support bar is $(x + 1)$ inches. What is the measure, in inches, of the vertical support bar?



Pythagorean Theorem Practice Name Key

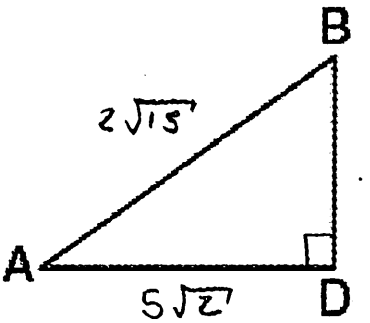
1. In the diagram below of ΔADB, $m\angle BDA = 90$, $AD = 5\sqrt{2}$, and $AB = 2\sqrt{15}$. What is the length of BD?

$(5\sqrt{2})^2 + b^2 = (2\sqrt{15})^2$

$50 + b^2 = 60$

$b^2 = 10$

$b = \sqrt{10}$ $\overline{BD} = \sqrt{10}$



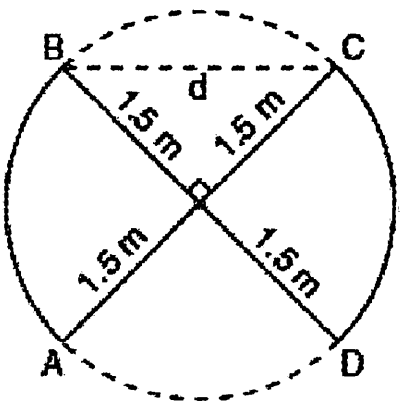
2. An overhead view of a revolving door is shown in the accompanying diagram. Each panel is 1.5 meters wide. What is the approximate width of d, the opening from B to C to the nearest hundredth of a meter?

$(1.5)^2 + (1.5)^2 = d^2$

$2.25 + 2.25 = d^2$

$4.5 = d^2$

$2.12\text{ m} = d$



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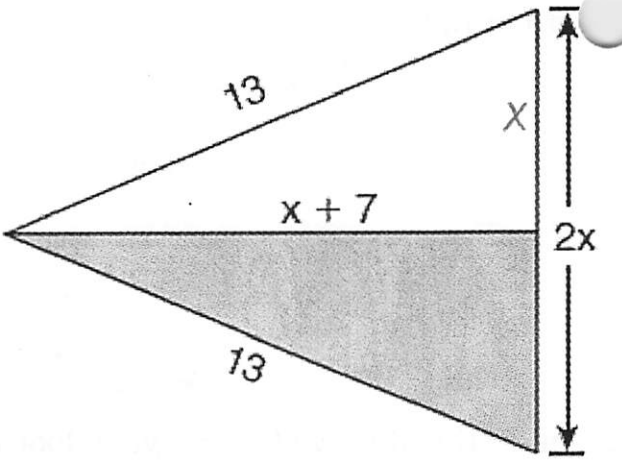

$x^2 + (x+7)^2 = 13^2$

$x^2 + x^2 + 14x + 49 = 169$

$2x^2 + 14x - 120 = 0$

$2(x^2 + 7x - 60) = 0$

$2(x+12)(x-5) = 0$

$2 \neq 0$ | $x + 12 = 0$ | $x - 5 = 0$

$x = -12$ | $x = 5$

Length of Base = 10

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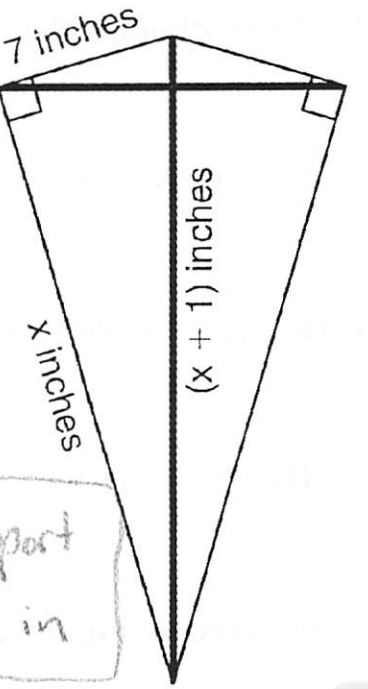

$7^2 + x^2 = (x+1)^2$

$49 + x^2 = x^2 + 2x + 1$

$49 = 2x + 1$

$-1 \quad -1$

$\frac{48}{2} = \frac{2x}{2}$

$24 = x$

Vertical Support Bar = 25 in