

INTEGRATED ALGEBRA

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

INTEGRATED ALGEBRA

Thursday, January 28, 2010 — 1:15 to 4:15 p.m., only

Student Name: Steve Watson

School Name: JMAP.ORG

Print your name and the name of your school on the lines above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

This examination has four parts, with a total of 39 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

The use of any communications device is strictly prohibited when taking this examination. If you use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

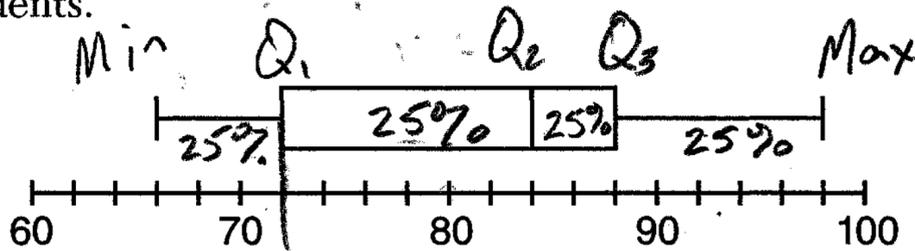
DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 30 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [60]

Use this space for computations.

- 1 The box-and-whisker plot below represents the math test scores of 20 students.



What percentage of the test scores are less than 72?

- (1) 25 (3) 75
 (2) 50 (4) 100

- 2 A bag contains eight green marbles, five white marbles, and two red marbles. What is the probability of drawing a red marble from the bag?

- (1) $\frac{1}{15}$ (3) $\frac{2}{13}$
 (2) $\frac{2}{15}$ (4) $\frac{13}{15}$

Green	8
White	5
Red	2
Total	15

$$P(\text{event}) = \frac{\# \text{ time event happens}}{\text{total possible outcomes}}$$

$$P(\text{red}) = \frac{2}{15}$$

- 3 Julia went to the movies and bought one jumbo popcorn and two chocolate chip cookies for \$5.00. Marvin went to the same movie and bought one jumbo popcorn and four chocolate chip cookies for \$6.00. How much does one chocolate chip cookie cost?

- (1) \$0.50 (3) \$1.00
 (2) \$0.75 (4) \$2.00

Popcorn	Cookies	Dollars
$1P$	$+ 2C$	$= 5$
$1P$	$+ 4C$	$= 6$
<hr/>		
	$2C$	$= 1$
	C	$= \frac{1}{2}$

Use this space for computations.

4 Given:

$Q = \{0, 2, 4, 6\}$ Not 3, Not 1

$W = \{0, 1, 2, 3\}$ Not 4

$Z = \{1, 2, 3, 4\}$ Not 0

What is the intersection of sets Q, W, and Z?

(1) $\{2\}$

(2) $\{0, 2\}$

(3) $\{1, 2, 3\}$

(4) $\{0, 1, 2, 3, 4, 6\}$

Z is the only # that appears in all 3 sets

5 Roger is having a picnic for 78 guests. He plans to serve each guest at least one hot dog. If each package, p , contains eight hot dogs, which inequality could be used to determine how many packages of hot dogs Roger will need to buy?

(1) $p \geq 78$

(2) $8p \geq 78$
at least

(3) $8 + p \geq 78$

(4) $78 - p \geq 8$

$\frac{78}{8} = P = 9 \frac{6}{8}$

$\frac{78}{8} = P$

$78 = 8P$
equal

6 In a science fiction novel, the main character found a mysterious rock that decreased in size each day. The table below shows the part of the rock that remained at noon on successive days.

Day	Fractional Part of the Rock Remaining
1	$\frac{1}{1}$
2	$\frac{1}{2}$
3	$\frac{1}{4}$
4	$\frac{1}{8}$

The denominator doubles every day

Day	Denominator
1	1
2	2
3	4
4	8
5	16
6	32
7	64

Which fractional part of the rock will remain at noon on day 7?

(1) $\frac{1}{128}$

(2) $\frac{1}{64}$

(3) $\frac{1}{14}$

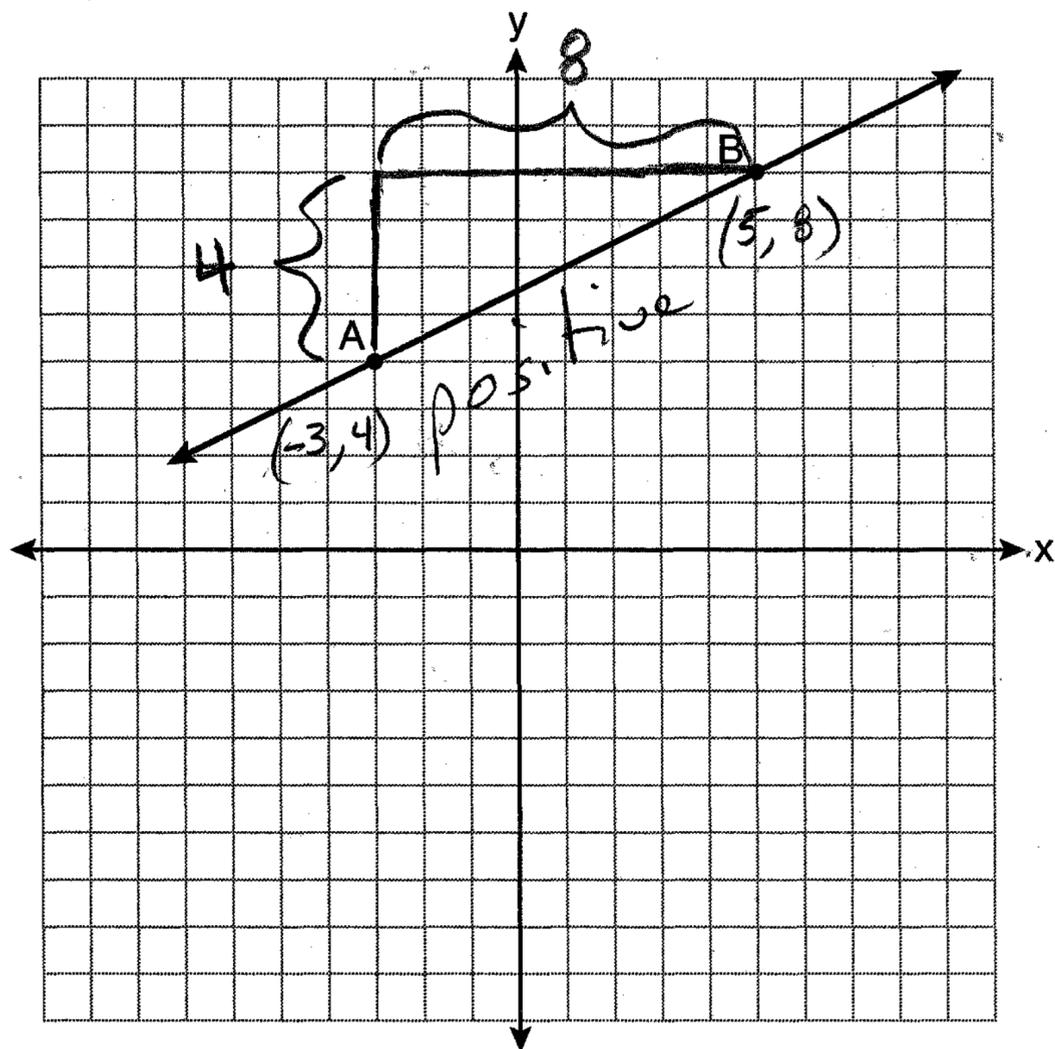
(4) $\frac{1}{12}$

- or -

$\frac{1}{2^{(n-1)}} \Rightarrow \frac{1}{2^6} \Rightarrow \frac{1}{64}$

7 In the diagram below, what is the slope of the line passing through points A and B?

Use this space for computations.



Slope = $m = \frac{\text{rise}}{\text{run}} = \frac{4}{8} = \frac{1}{2}$
- or -
 $m = \frac{y_2 - y_1}{x_2 - x_1}$
 $m = \frac{8 - 4}{5 - (-3)} = \frac{4}{8} = \frac{1}{2}$

(1) -2

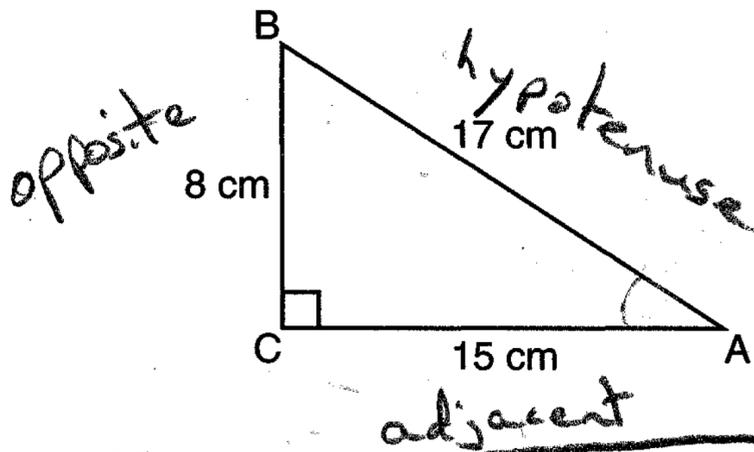
(3) $-\frac{1}{2}$

(2) 2

(4) $\frac{1}{2}$

Use this space for computations.

8 Which equation shows a correct trigonometric ratio for angle A in the right triangle below?



SOH-CAH-TOA

$$\sin = \frac{\text{opp}}{\text{hyp}} = \frac{8}{17}$$

$$\cos = \frac{\text{adj}}{\text{hyp}} = \frac{15}{17}$$

$$\tan = \frac{\text{opp}}{\text{adj}} = \frac{8}{15}$$

- ~~(1) $\sin A = \frac{15}{17}$~~
- ~~(2) $\tan A = \frac{8}{17}$~~

- (3) $\cos A = \frac{15}{17}$
- ~~(4) $\tan A = \frac{15}{8}$~~

9 Debbie solved the linear equation $3(x + 4) - 2 = 16$ as follows:

[Line 1] $3(x + 4) - 2 = 16$

[Line 2] $3(x + 4) = 18$

[Line 3] $3x + 4 = 18 \neq$

[Line 4] $3x = 14$

[Line 5] $x = 4\frac{2}{3}$

$3(x + 4) = 18$

$3x + 12 = 18$

Should be 12, not 4

She made an error between lines

- (1) 1 and 2
- (2) 2 and 3
- (3) 3 and 4
- (4) 4 and 5

10 The value of the expression $-|a - b|$ when $a = 7$ and $b = -3$ is

Use this space for computations.

(1) -10

(3) -4

(2) 10

(4) 4

$$\begin{aligned} & -|a - b| \\ & -|7 - (-3)| \\ & -|7 + 3| \end{aligned} \quad \begin{array}{l} \rightarrow -|10| \\ -10 \end{array}$$

11 Which expression represents $\frac{12x^3 - 6x^2 + 2x}{2x}$ in simplest form?

(1) $6x^2 - 3x$

(3) $6x^2 - 3x + 1$

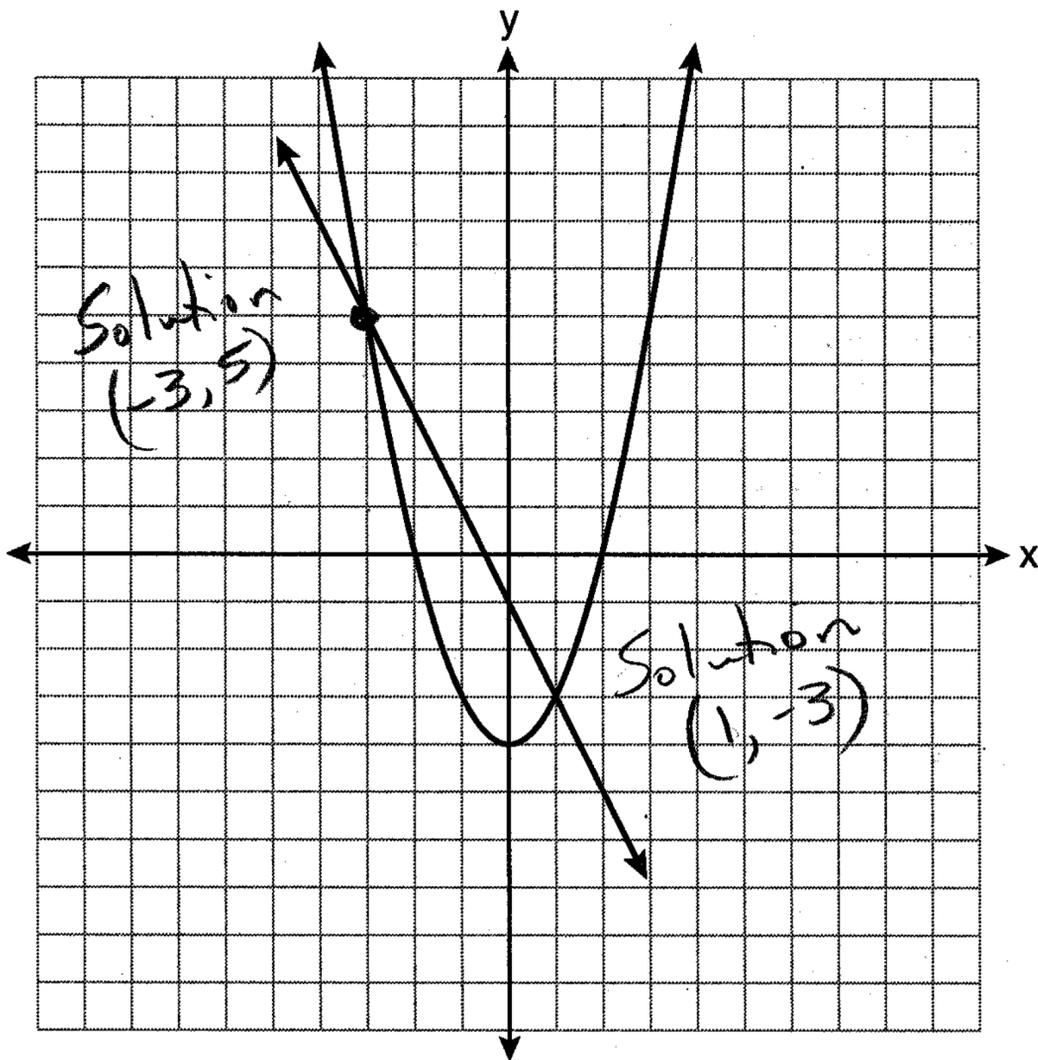
(2) $10x^2 - 4x$

(4) $10x^2 - 4x + 1$

$$\frac{12x^3}{2x} - \frac{6x^2}{2x} + \frac{2x}{2x}$$

$6x^2 - 3x + 1$

12 Which ordered pair is a solution of the system of equations shown in the graph below?



(1) (-3, 1)

(3) (0, -1)

(2) (-3, 5)

(4) (0, -4)

Use this space for computations.

13 Which equation represents the line that passes through the points $(-3,7)$ and $(3,3)$?

~~(1) $y = \frac{2}{3}x + 1$~~

(3) $y = -\frac{2}{3}x + 5$

~~(2) $y = \frac{2}{3}x + 9$~~

(4) $y = -\frac{2}{3}x + 9$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{3 - 7}{3 - (-3)} = \frac{-4}{6} = -\frac{2}{3}$$

$$y = mx + b$$

$$y = -\frac{2}{3}x + b$$

$(3, 3)$ $3 = -\frac{2}{3}(\frac{3}{1}) + b$

$$3 = -2 + b$$

$$5 = b$$

$$y = -\frac{2}{3}x + 5$$

14 Which data table represents univariate data?

Side Length of a Square	Area of Square
2	4
3	9
4	16
5	25

(1)

Age Group	Frequency
20-29	9
30-39	7
40-49	10
50-59	4

(3)

Hours Worked	Pay
20	\$160
25	\$200
30	\$240
35	\$280

(2)

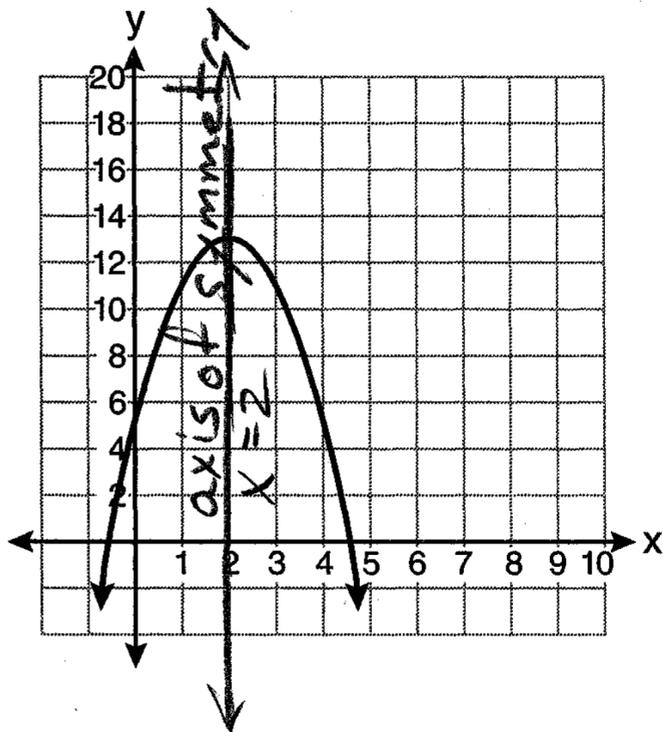
People	Number of Fingers
2	20
3	30
4	40
5	50

(4)

univariate
one variable

15 What is the equation of the axis of symmetry of the parabola shown in the diagram below?

Use this space for computations.



(1) $x = -0.5$

(2) $x = 2$

(3) $x = 4.5$

(4) $x = 13$

16 The members of the senior class are planning a dance. They use the equation $r = pn$ to determine the total receipts. What is n expressed in terms of r and p ?

(1) $n = r + p$

(3) $n = \frac{p}{r}$

(2) $n = r - p$

(4) $n = \frac{r}{p}$

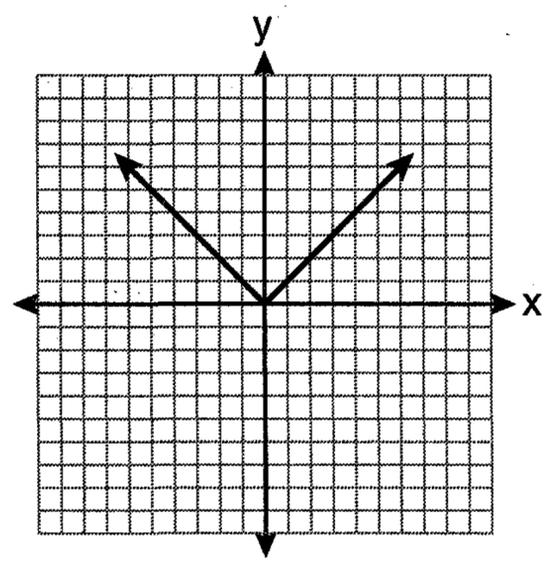
$$r = pn$$

$$\frac{r}{p} = n$$

$a = 1$, which is positive
Graph opens up.

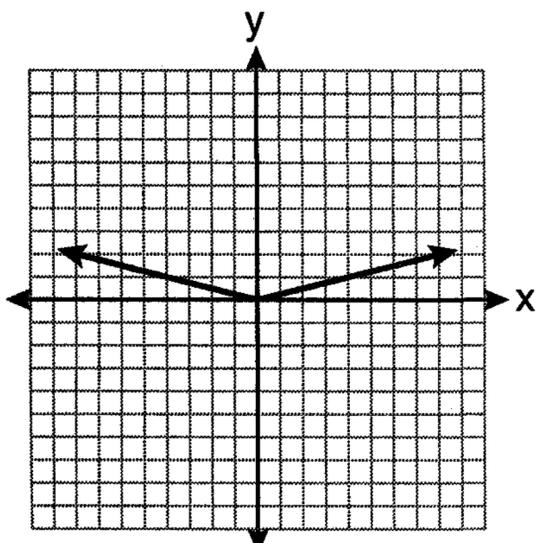
17 The graph of the equation $y = |x|$ is shown in the diagram below.

Use this space for computations.

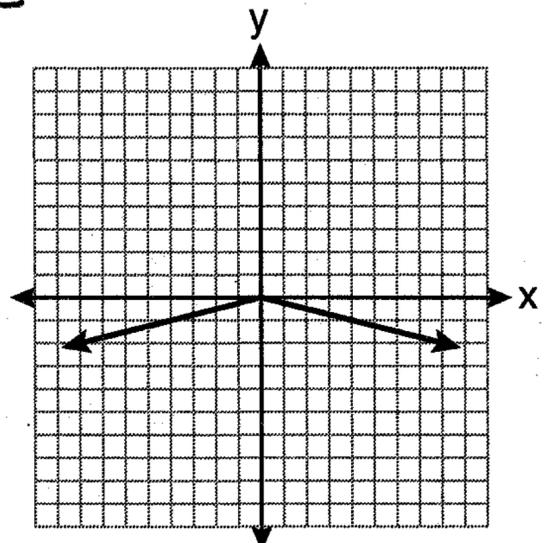


Which diagram could represent a graph of the equation $y = a|x|$ when $-1 < a < 0$?

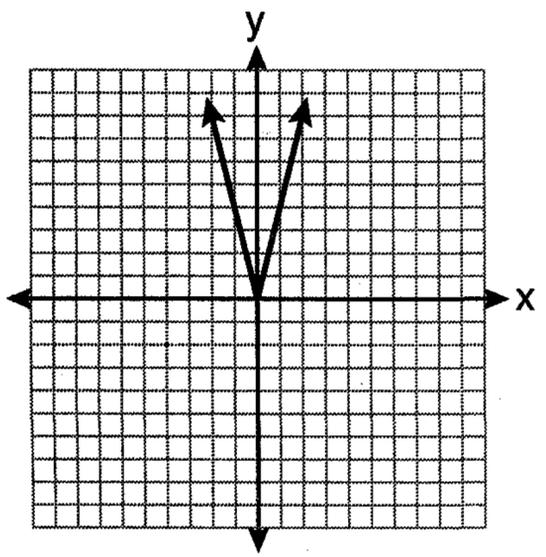
a is negative. Graph must open down



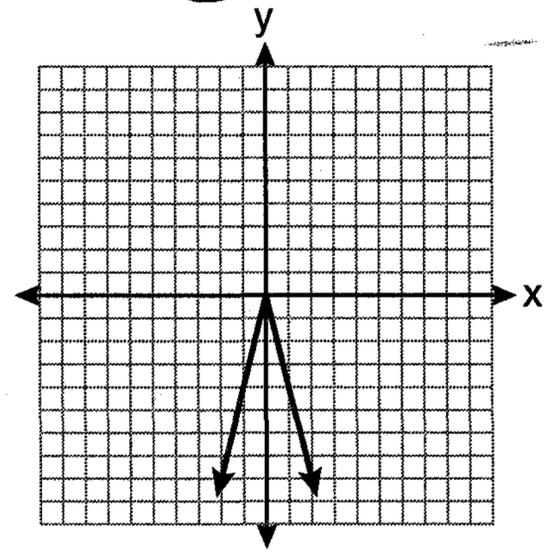
(1)



(3)



(2)



(4)

Is the graph fat or skinny?
 $-1 < a < 0$

This is a negative fraction between -1 and 0. As a approaches zero, the graph gets fatter.

[OVER]

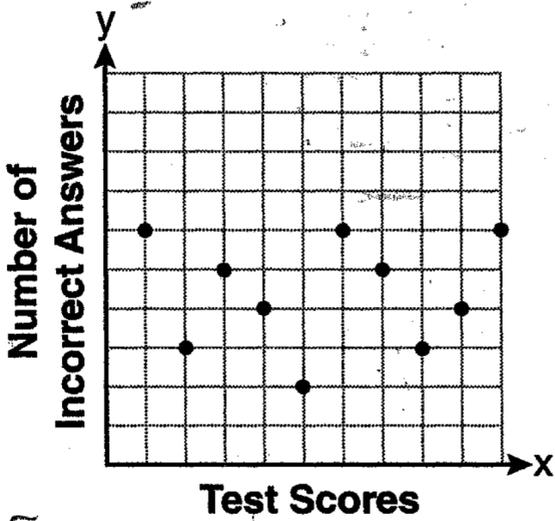
Use this space for computations.

18 Which relation represents a function?

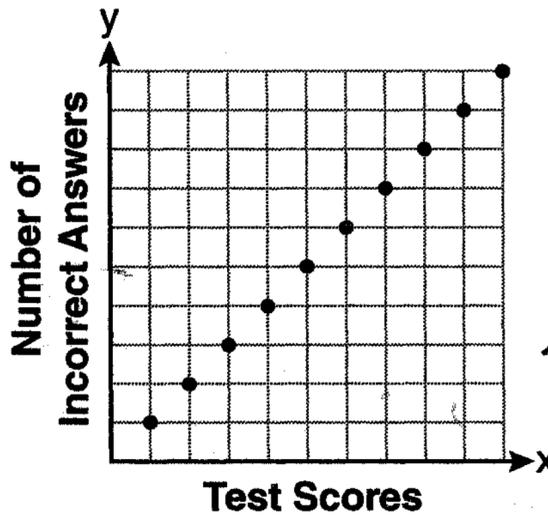
- (1) $\{(0,3), (2,4), (0,6)\}$ $x=0$ has 2 y 's
- (2) $\{(-7,5), (-7,1), (-10,3), (-4,3)\}$ $x=-7$ has 2 y 's
- (3) $\{(2,0), (6,2), (6,-2)\}$ $x=6$ has 2 y 's
- (4) $\{(-6,5), (-3,2), (1,2), (6,5)\}$

A function has one and only one value of y for every value of x .

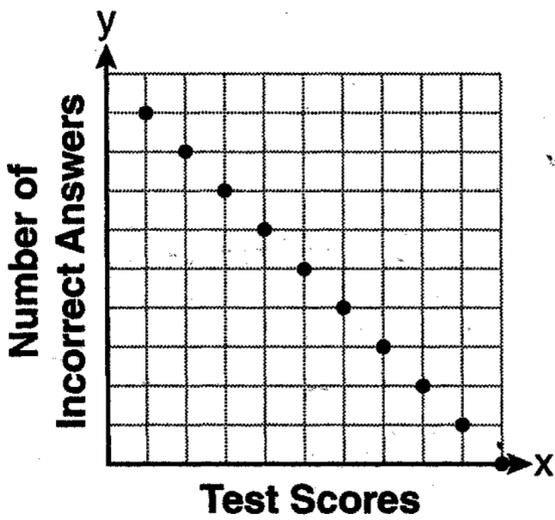
19 Which scatter plot shows the relationship between x and y if x represents a student score on a test and y represents the number of incorrect answers a student received on the same test?



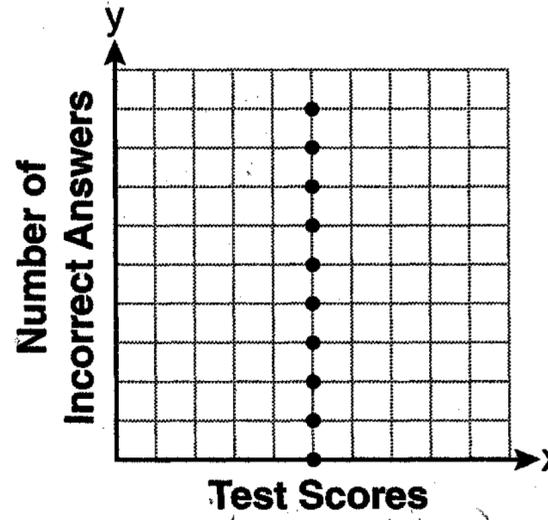
(1)



(3)



(2)



(4)

Positive Correlation
Test scores would go up with the # of correct answers. Problem specifies incorrect answers.

No relation

There is a negative correlation between the number of incorrect answers and the test score.

Underlined

Use this space for computations.

20 Which expression is equivalent to $3^3 \cdot 3^4$?

- (1) 9^{12}
- (2) 9^7

(3) 3^{12}

(4) 3^7

$3^3 \cdot 3^4 = 3^{(3+4)} = 3^7$

Check on Calculator

$3 \wedge 3 * 3 \wedge 4 = 2187$

$3 \wedge 7 = 2187$

21 Which point is on the line $4y - 2x = 0$?

(1) $(-2, -1)$

(2) $(-2, 1)$

(3) $(-1, -2)$

(4) $(1, 2)$

	$4y - 2x = 0$
$(-2, -1)$	$4(-1) - 2(-2) = 0$ ✓
$(-2, 1)$	$4(1) - 2(-2) = 0$
$(-1, -2)$	$4(-2) - 2(-1) = 0$
$(1, 2)$	$4(2) - 2(1) = 0$

22 If Ann correctly factors an expression that is the difference of two perfect squares, her factors could be

(1) $(2x + y)(x - 2y)$

(2) $(2x + 3y)(2x - 3y)$

(3) $(x - 4)(x - 4)$

(4) $(2y - 5)(y - 5)$

$(a^2 + b^2) = (a + b)(a - b)$

one is positive +
the other
is negative

23 Which ordered pair is in the solution set of the following system of linear inequalities?

$y < 2x + 2$

$y \geq -x - 1$

(1) $(0, 3)$

(2) $(2, 0)$

(3) $(-1, 0)$

(4) $(-1, -4)$

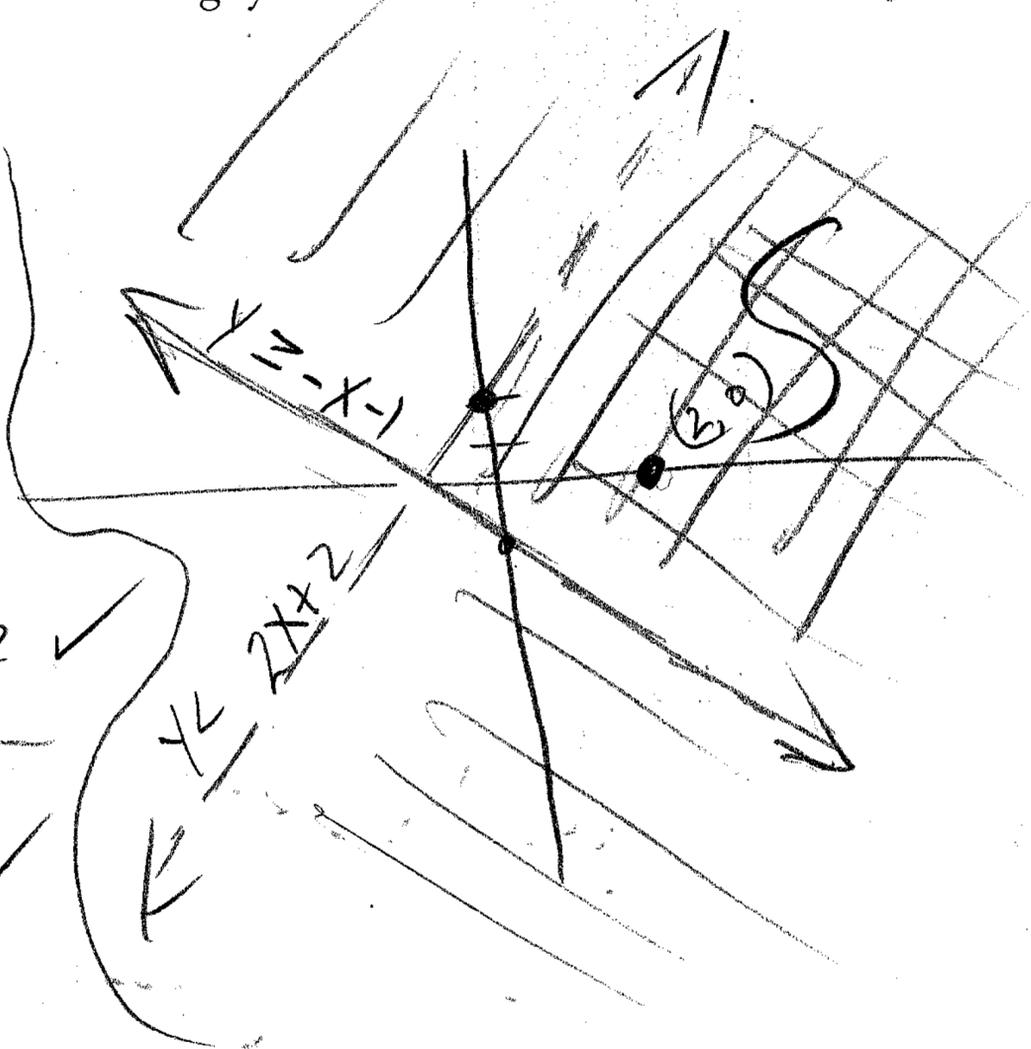
Check $(2, 0)$

$y < 2x + 2$

$0 < 2(2) + 2$ ✓
 $0 < 6$

$y \geq -x - 1$

$0 \geq -(2) - 1$ ✓
 $0 \geq -3$ ✓



Use this space for computations.

24 The expression $6\sqrt{50} + 6\sqrt{2}$ written in simplest radical form is

(1) $6\sqrt{52}$

(2) $12\sqrt{52}$

(3) $17\sqrt{2}$

(4) $36\sqrt{2}$

$$\begin{aligned}
 &6\sqrt{50} + 6\sqrt{2} \\
 &6\sqrt{25 \cdot 2} + 6\sqrt{2} \\
 &6 \cdot 5\sqrt{2} + 6\sqrt{2} \\
 &30\sqrt{2} + 6\sqrt{2} \\
 &36\sqrt{2}
 \end{aligned}$$

25 What is the sum of $\frac{3x^2}{x-2}$ and $\frac{x^2}{x-2}$?

(1) $\frac{3x^4}{(x-2)^2}$

(2) $\frac{3x^4}{x-2}$

(3) $\frac{4x^2}{(x-2)^2}$

(4) $\frac{4x^2}{x-2}$

If the denominators are the same, you can add the numerators.

26 Which equation represents a line parallel to the graph of $2x - 4y = 16$?

(1) $y = \frac{1}{2}x - 5$

(2) $y = -\frac{1}{2}x + 4$

(3) $y = -2x + 6$

(4) $y = 2x + 8$

same slope, different y intercept

$$2x - 4y = 16$$

$$-4y = -2x + 16$$

$$y = \frac{-2}{-4}x + \frac{16}{-4}$$

$$y = \frac{1}{2}x - 4$$

$$m = \frac{1}{2}$$

27 An example of an algebraic expression is

(1) $\frac{2x+3}{7} = \frac{13}{x}$

(2) $(2x+1)(x-7)$

(3) $4x - 1 = 4$

(4) $x = 2$

The other 3 choices are equations because they have equal signs.

Use this space for computations.

28 What is the solution set of $\frac{x+2}{x-2} = \frac{-3}{x}$?

- (1) $\{-2, 3\}$
- (2) $\{-3, -2\}$
- (3) $\{-1, 6\}$
- (4) $\{-6, 1\}$

$$\frac{x+2}{x-2} = \frac{-3}{x}$$

cross multiply

$$x(x+2) = (-3)(x-2)$$

$$x^2 + 2x = -3x + 6$$

$$x^2 + 5x - 6 = 0$$

$$(x+6)(x-1) = 0$$

$$x+6=0$$

$$x=-6$$

$$x-1=0$$

$$x=1$$

29 How many square inches of wrapping paper are needed to entirely cover a box that is 2 inches by 3 inches by 4 inches?

- (1) 18
- (2) 24
- (3) 26
- (4) 52

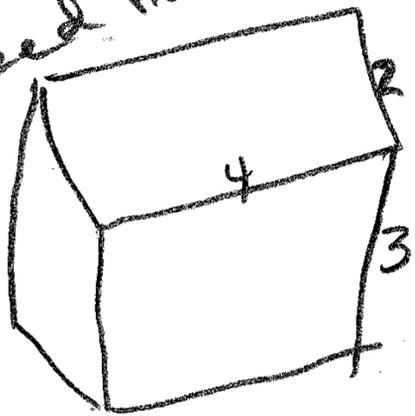
See below

30 Which situation describes a correlation that is *not* a causal relationship?

- (1) the length of the edge of a cube and the volume of the cube
- (2) the distance traveled and the time spent driving
- (3) the age of a child and the number of siblings the child has
- (4) the number of classes taught in a school and the number of teachers employed

The oldest child + the youngest child could have the same # of siblings.

There are 6 sides that need to be covered.



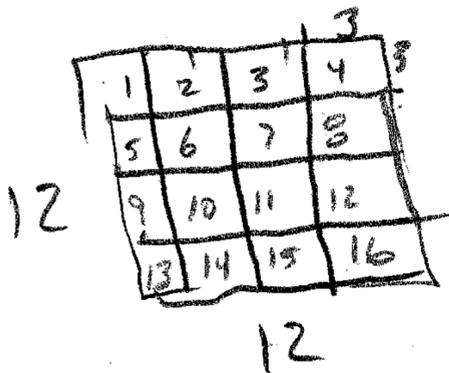
	Area	For One	For Two
Top + Bottom	2×4	8	16
Front + Back	4×3	12	24
Left + Right	2×3	6	12
Total			52

Part II

Answer all 3 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [6]

31 Angela wants to purchase carpeting for her living room. The dimensions of her living room are 12 feet by 12 feet. If carpeting is sold by the square yard, determine how many square yards of carpeting she must purchase.

3 feet = 1 yard
9 square feet = 1 square yard

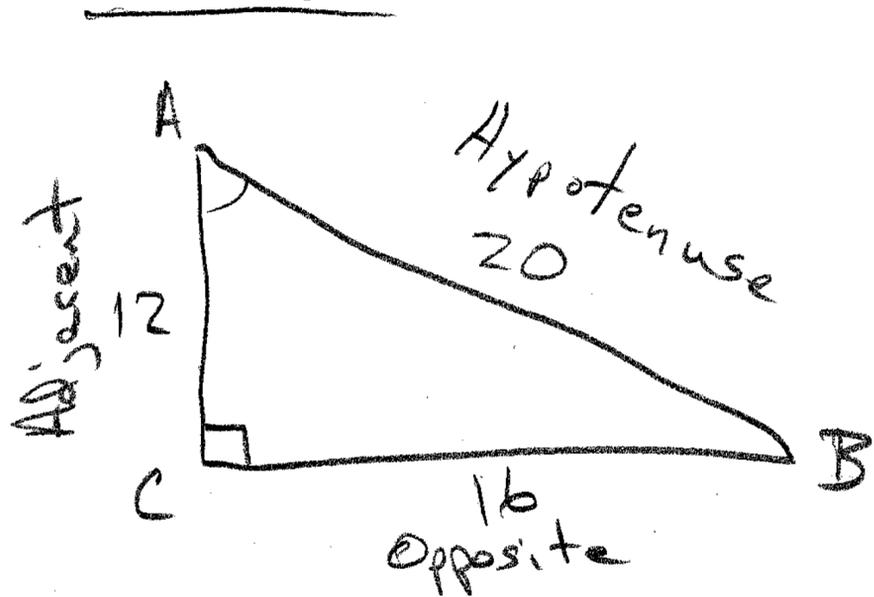


$$\text{Area} = 12 \times 12 = 144 \text{ sq. ft.}$$

$$\frac{144}{9} = 16 \text{ sq. yards}$$

32 In right triangle ABC , $AB = 20$, $AC = 12$, $BC = 16$, and $m\angle C = 90$.

Find, to the nearest degree, the measure of $\angle A$.



$$m\angle A = 53^\circ$$

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$$\text{or } \sin A = \frac{\text{opp}}{\text{hyp}} = \frac{16}{20}$$

$$\arcsin \frac{16}{20} = 53^\circ$$

$$\text{or } \cos A = \frac{\text{adj}}{\text{hyp}} = \frac{12}{20}$$

$$\arccos \frac{12}{20} = 53^\circ$$

$$\text{or } \tan A = \frac{\text{opp}}{\text{adj}} = \frac{16}{12}$$

$$\arctan \frac{16}{12} = 53^\circ$$

33 Jon is buying tickets for himself for two concerts. For the jazz concert, 4 tickets are available in the front row, and 32 tickets are available in the other rows. For the orchestra concert, 3 tickets are available in the front row, and 23 tickets are available in the other rows. Jon is randomly assigned one ticket for each concert.

Determine the concert for which he is more likely to get a front-row ticket. Justify your answer.

$$\text{Jazz} \quad \frac{4 \text{ front row}}{4+32 \text{ total}} = \frac{4}{36} = \frac{1}{9} = .\overline{1111}$$

$$P(\text{event}) = \frac{\# \text{ desirable outcomes}}{\text{total possible outcomes}}$$

$$P(\text{front row}) = \frac{1}{9}$$

Orchestra

$$\frac{3 \text{ front row}}{3+23 \text{ total}} = \frac{3}{26} = .1153\dots$$

The probability of getting a front row seat is higher in the orchestra concert.

Part III

Answer all 3 questions in this part. Each correct answer will receive 3 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [9]

34 Find the roots of the equation $x^2 - x = 6$ algebraically.

$$x^2 - x = 6$$

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

$$x-3 = 0$$

$$x+2 = 0$$

$$x = 3$$

$$x = -2$$

$$\{3, -2\}$$

35 Ms. Mosher recorded the math test scores of six students in the table below.

Student	Student Score
Andrew	72
John	80
George	85
Amber	93
Betty	78
Roberto	80

Determine the mean of the student scores, to the nearest tenth.

$$\frac{\text{sum}}{\text{count}} = \frac{488}{6} = \boxed{81.3}$$

Determine the median of the student scores.

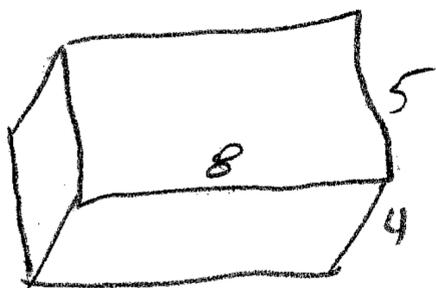
$$72, 78, 80, 80, 85, 93$$

↑ middle → $\frac{80+80}{2} = \boxed{80}$

Describe the effect on the mean and the median if Ms. Mosher adds 5 bonus points to each of the six students' scores.

Both the mean and the median will increase by 5

36 Using his ruler, Howell measured the sides of a rectangular prism to be 5 cm by 8 cm by 4 cm. The actual measurements are 5.3 cm by 8.2 cm by 4.1 cm. Find Howell's relative error in calculating the volume of the prism, to the nearest thousandth.

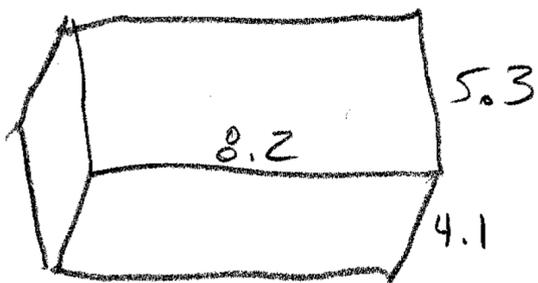


Measured

$$V = lwh$$

$$V = (5)(8)(4) = 160$$

Actual



$$V = lwh$$

$$V = (5.3)(8.2)(4.1) = 178.186$$

$$\text{Error} = \frac{\text{Actual} - \text{Measured}}{\text{Actual}}$$

$$\frac{178.186 - 160}{178.186}$$

$$178.186$$

$$\frac{18.186}{178.186}$$

$$178.186$$

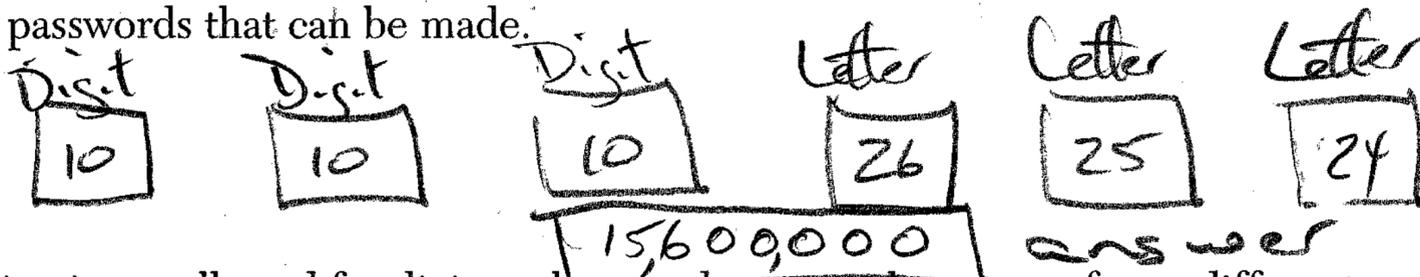
$$\boxed{0.102}$$

Part IV

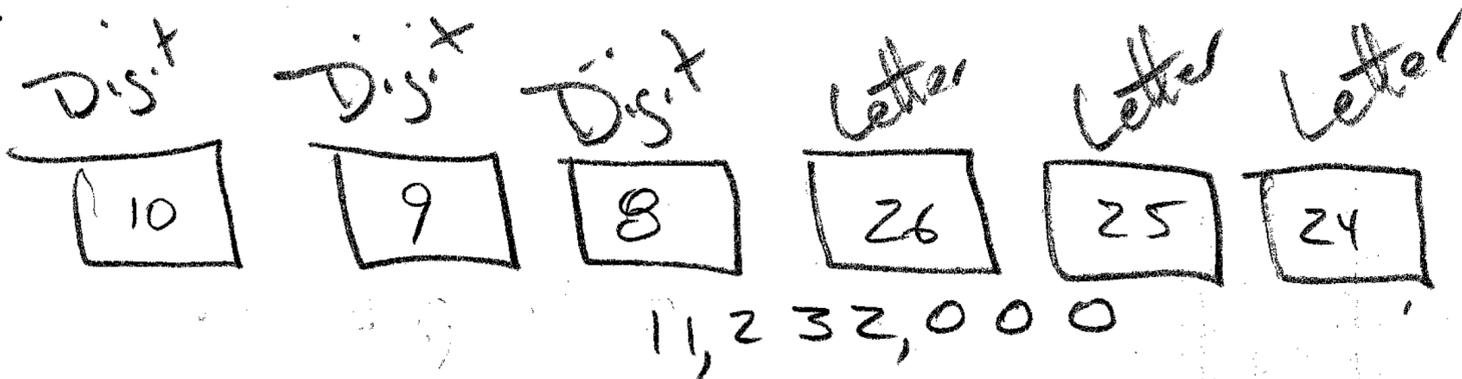
Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

37 A password consists of three digits, 0 through 9, followed by three letters from an alphabet having 26 letters.

If repetition of digits is allowed, but repetition of letters is not allowed, determine the number of different passwords that can be made.



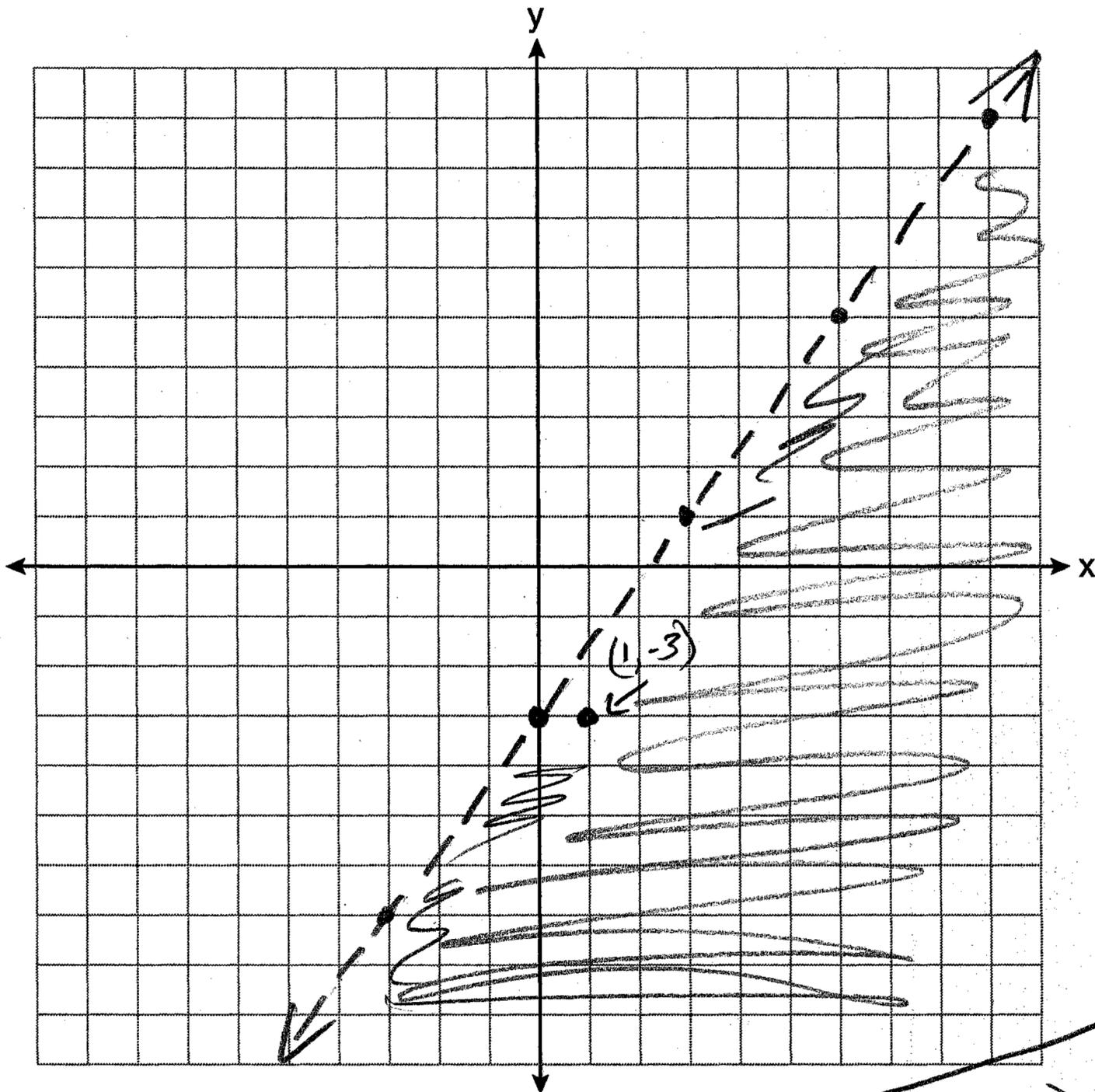
If repetition is not allowed for digits or letters, determine how many fewer different passwords can be made.



$$\begin{array}{r}
 15,600,000 \\
 - 11,232,000 \\
 \hline
 4,368,000
 \end{array}$$

38 Graph the solution set for the inequality $4x - 3y > 9$ on the set of axes below.

Determine if the point $(1, -3)$ is in the solution set. Justify your answer.



$$4x - 3y > 9$$

$$4x - 3y > 9$$

$$-3y > -4x + 9$$

Sign Change → $y < \frac{-4x}{-3} + \frac{9}{-3}$

$$y < \frac{4}{3}x - 3$$

Point $(1, -3)$ is in the solution set

39 Find three consecutive positive even integers such that the product of the second and third integers is twenty more than ten times the first integer. [Only an algebraic solution can receive full credit.]

$$\begin{aligned}x &= \text{first integer} \\x+2 &= \text{second integer} \\x+4 &= \text{third integer}\end{aligned}$$

$$(x+2)(x+4) = 10x + 20$$

$$x^2 + 4x + 2x + 8 = 10x + 20$$

$$x^2 + 6x - 10x - 12 = 0$$

$$x^2 - 4x - 12 = 0$$

$$(x+2)(x-6) = 0$$

$$x+2 = 0$$

$$x = -2$$

Not a positive
even integer

$$x-6 = 0$$

$$\underline{x = 6}$$

First Integer = 6
Second Integer = 8
Third Integer = 10