Dividing Fractions by Fractions

Reciprocals and Division   🔊 Two numbers are **reciprocals** if their product is 1.

If a nonzero number is named as a fraction, $\frac{a}{b}$, then its reciprocal is $\frac{b}{a}$.

The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

The reciprocal of 4, or $\frac{4}{1}$, is $\frac{1}{4}$.

Show that the **product** of the following and their **reciprocals** is 1.

1. $\frac{3}{4} \cdot =$

2. $\frac{1}{7} \cdot =$

3. $5\frac{1}{2} \cdot =$

▽ **Rule for Dividing With Fractions**   ◉ To divide a nonzero number, including by a fraction, you can multiply by the divisor's reciprocal.

Example: $2 \div \frac{3}{4} = 2 \times \frac{4}{3}$

$$= \frac{2 \times 4}{3}$$

$$= \frac{8}{3}, \text{ or } 2\frac{2}{3}$$

1. $5 \div \frac{1}{4} =$

2. $\frac{2}{7} \div 6 =$

3. $\frac{2}{3} \div \frac{1}{12} =$

4. $\frac{3}{4} \div \frac{7}{8} =$

Dividing Mixed Numbers

You can extend what you know about dividing whole numbers and fractions to divide <u>mixed numbers</u>.

Find $2\frac{2}{5} \div \frac{4}{5}$.

$$2\frac{2}{5} \div \frac{4}{5} = \frac{12}{5} \div \frac{4}{5}$$

Change any mixed number to an improper fraction.

$$= \frac{12}{5} \times \frac{5}{4}$$

Change dividing to multiplying by the reciprocal.

$$= \frac{3\cancel{12}}{\cancel{5}_1} \times \frac{\cancel{5}^1}{\cancel{4}_1}$$

$$= \frac{3}{1}, \text{ or } 3$$

1. $1\frac{5}{9} \div \frac{8}{9}$

2. $6\frac{3}{4} \div 3$

3. $\frac{5}{6} \div 1\frac{1}{4}$

4. $1\frac{1}{15} \div 1\frac{3}{5}$

5. $2\frac{1}{2} \div 1\frac{3}{7}$

6. $1\frac{2}{7} \div \frac{4}{7}$

Dividing Fractions by Fractions

Multiplicative inverse

Reciprocals and Division 🔊 Two numbers are **reciprocals** if their product is 1.

If a nonzero number is named as a fraction, $\frac{a}{b}$, then its reciprocal is $\frac{b}{a}$.

→ The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

→ The reciprocal of 4, or $\frac{4}{1}$, is $\frac{1}{4}$.

Show that the **product** of the following and their **reciprocals** is 1.

1. $\frac{3}{4} \cdot \frac{4}{3} = \frac{12}{12} = 1$

2. $\frac{1}{7} \cdot \frac{7}{1} = \frac{7}{7} = 1$

3. $5\frac{+1}{\times 2} \cdot \frac{2}{11} =$

$\frac{11}{2} \cdot \frac{2}{11} = \frac{22}{22} = 1$

▽ **Rule for Dividing With Fractions** ⦿ To divide a nonzero number, including by a fraction, you can multiply by the divisor's reciprocal.

Example: $2 \div \frac{3}{4} = 2 \times \frac{4}{3}$

$= \frac{2 \times 4}{3}$

$= \frac{8}{3}$, or $2\frac{2}{3}$

1. $5 \div \frac{1}{4} = \frac{5}{1} \div \frac{1}{4}$

$\frac{5}{1} \cdot \frac{4}{1} = \frac{20}{1} = \boxed{20}$

2. $\frac{2}{7} \div 6 = \frac{2}{7} \div \frac{6}{1}$

$\frac{2}{7} \cdot \frac{1}{6_3} = \boxed{\frac{1}{21}}$

3. $\frac{2}{3} \div \frac{1}{12} = \frac{2}{3} \cdot \frac{12^4}{1} = \frac{8}{1} = \boxed{8}$

4. $\frac{3}{4} \div \frac{7}{8} = \frac{3}{4} \cdot \frac{8^2}{7} = \boxed{\frac{6}{7}}$

Dividing Mixed Numbers

🔊 You can extend what you know about dividing whole numbers and fractions to divide <u>mixed numbers</u>.

Find $2\frac{2}{5} \div \frac{4}{5}$.

$$2\frac{2}{5} \div \frac{4}{5} = \frac{12}{5} \div \frac{4}{5}$$

> Change any mixed number to an improper fraction.

$$= \frac{12}{5} \times \frac{5}{4}$$

> Change dividing to multiplying by the reciprocal.

$$= \frac{\overset{3}{\cancel{12}}}{\underset{1}{\cancel{5}}} \times \frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{4}}}$$

$$= \frac{3}{1}, \text{ or } 3$$

1. $1\frac{\overset{+5}{\times 9}}{} \div \frac{8}{9}$

$$\frac{14}{9} \div \frac{8}{9}$$

$$\frac{\overset{7}{\cancel{14}}}{\underset{1}{\cancel{9}}} \cdot \frac{\overset{1}{\cancel{9}}}{\underset{4}{\cancel{8}}} = \frac{7}{4} = \boxed{1\frac{3}{4}}$$

2. $6\frac{\overset{+3}{\times 4}}{} \div 3$

$$\frac{27}{4} \div \frac{3}{1}$$

$$\frac{\overset{9}{\cancel{27}}}{4} \cdot \frac{1}{\underset{1}{\cancel{3}}} = \frac{9}{4} = \boxed{2\frac{1}{4}}$$

3. $\frac{5}{6} \div 1\frac{\overset{+1}{\times 4}}{}$

$$\frac{5}{6} \div \frac{5}{4}$$

$$\frac{\overset{1}{\cancel{5}}}{\underset{3}{\cancel{6}}} \cdot \frac{\overset{2}{\cancel{4}}}{\underset{1}{\cancel{5}}} = \boxed{\frac{2}{3}}$$

4. $1\frac{1}{15} \div 1\frac{3}{5}$

$$\frac{16}{15} \div \frac{8}{5}$$

$$\frac{\overset{2}{\cancel{16}}}{\underset{3}{\cancel{15}}} \cdot \frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{8}}} = \boxed{\frac{2}{3}}$$

5. $2\frac{\overset{+1}{\times 2}}{} \div 1\frac{\overset{+3}{\times 7}}{}$

$$\frac{5}{2} \div \frac{10}{7}$$

$$\frac{\overset{1}{\cancel{5}}}{2} \cdot \frac{7}{\underset{2}{\cancel{10}}} = \frac{7}{4} = \boxed{1\frac{3}{4}}$$

6. $1\frac{\overset{+2}{\times 7}}{} \div \frac{4}{7}$

$$\frac{9}{7} \div \frac{4}{7}$$

$$\frac{9}{\underset{1}{\cancel{7}}} \cdot \frac{\overset{1}{\cancel{7}}}{4} = \frac{9}{4} = \boxed{2\frac{1}{4}}$$