Volume, Lateral Area, and Surface Area of Cylinders, Cones and Spheres

Find the Volume of the figures to the nearest whole number.

1. \( \text{Volume of cone} \)

2. \( \text{Volume of sphere} \)

Find the Lateral Area of the figures to the nearest whole number.

3. \( \text{Lateral area of cylinder} \)

4. \( \text{Lateral area of cone} \)

Find the Surface Area of the figures to the nearest whole number.

5. \( \text{Surface area of cylinder} \)

6. \( \text{Surface area of sphere} \)
For you to Practice

1. Find the surface area of the cylinder below:

2. a. Find the slant height of the cone given the altitude and the radius.
   b. Find the surface area of the cone to the nearest tenth.

3. A storage container in the shape of a right circular cylinder is shown in the accompanying diagram.

   What is the volume of this container, to the nearest hundredth?
   [C] 251.33 in$^3$  [D] 125.66 in$^3$

4. If the surface area of a sphere is represented by $144\pi$, what is the volume in terms of $\pi$?
In the accompanying diagram, a rectangular container with the dimensions 10 inches by 16 inches by 20 inches is to be filled with water, using a cylindrical cup whose radius is 2 inches and whose height is 5 inches. What is the maximum number of full cups of water that can be placed into the container without the water overflowing the container?

2. Students in one mathematics class noticed that a local movie theater sold popcorn in different shapes of containers, for different prices. They wondered which of the choices was the best buy. Analyze the three popcorn containers below. Which is the best buy? Explain.
3. A sphere has a volume of $7776\pi$ cubic inches. Find the surface area of the sphere.

4. A right circular cylinder has an altitude of 11 feet and a radius of 5 feet. What is the lateral area, in square feet, of the cylinder, to the nearest tenth?

5. A right circular cylinder has a volume of 1,000 cubic inches and a height of 8 inches. What is the radius of the cylinder to the nearest tenth of an inch?

6. Find the difference in the volumes of the cones created by rotating the triangle shown below around the $x$-axis and around the $y$-axis. Write your answer in terms of $\pi$. 

![Diagram of a triangle with coordinates (0,0), (3,2), and (0,2)]
Volume, Lateral Area, and Surface Area of Cylinders, Cones and Spheres

Find the Volume of the figures to the nearest whole number.

1. \( V = \frac{1}{3} \pi s^2 h \)
   \[ V = \frac{1}{3} \pi (5)^2 (12) \]
   \[ V = 314 \text{ in}^3 \]

2. \( V = \frac{4}{3} \pi r^3 \)
   \[ V = \frac{4}{3} \pi (7)^3 \]
   \[ V = 1437 \text{ mi}^3 \]

Find the Lateral Area of the figures to the nearest whole number.

3. \( LA = 2\pi r h \)
   \[ LA = 2\pi (5.5)(10) \]
   \[ LA = 553 \text{ ft}^2 \]

4. \( LA = \pi r \ell \)
   \[ LA = \pi (6)(10) \]
   \[ LA = 188 \text{ ft}^2 \]

Find the Surface Area of the figures to the nearest whole number.

5. \( SA = LA + 2B \)
   \[ SA = 2\pi (3.5)(10) + 2\pi (3.5)^2 \]
   \[ SA = 297 \text{ cm}^2 \]

6. \( SA = 4\pi r^2 \)
   \[ SA = 4\pi (14)^2 \]
   \[ SA = 2463 \text{ in}^2 \]
For you to Practice

1. Find the surface area of the cylinder below:

   ![Cylinder Diagram]

   
   \[
   SA = 2\pi rh + 2\pi r^2
   \]
   
   \[
   SA = 2\pi (3.5)(5) + 2\pi (3.5)^2
   \]
   
   \[
   SA = 186.9 \text{ in}^2
   \]

2. a. Find the slant height of the cone given the altitude and the radius.

   \[
   \ell = \sqrt{h^2 + r^2}
   \]

   \[
   \ell = \sqrt{13^2 + 5^2}
   \]

   \[
   \ell = 13 \text{ in}
   \]

   b. Find the surface area of the cone to the nearest tenth.

   \[
   SA = \pi r\ell + \pi r^2
   \]

   \[
   SA = \pi (5)(13) + \pi (5)^2
   \]

   \[
   SA = 282.7
   \]

3. A storage container in the shape of a right circular cylinder is shown in the accompanying diagram.

   ![Cylinder Diagram]

   What is the volume of this container, to the nearest hundredth?

   [A] 502.65 in³  [B] 565.35 in³
   [C] 251.33 in³  [D] 125.66 in³

   \[
   V = \pi r^2 h
   \]

   \[
   V = \pi (4)^2 (10)
   \]

   \[
   V = 502.65 \text{ in}^3
   \]

4. If the surface area of a sphere is represented by \(144\pi\), what is the volume in terms of \(\pi\)?

   \[
   \frac{144\pi}{4\pi} = \frac{4\pi r^2}{4\pi}
   \]

   \[
   36 = r^2
   \]

   \[
   r = 6
   \]

   \[
   V = \frac{4}{3} \pi r^3
   \]

   \[
   V = \frac{4}{3} \pi (6)^3
   \]

   \[
   V = 288\pi
   \]
In the accompanying diagram, a rectangular container with the dimensions 10 inches by 15 inches by 20 inches is to be filled with water, using a cylindrical cup whose radius is 2 inches and whose height is 5 inches. What is the maximum number of full cups of water that can be placed into the container without the water overflowing the container?

\[ V = L \cdot W \cdot H \]
\[ V = 15 \cdot 10 \cdot 20 \]
\[ V = 3000 \text{ in}^3 \]

\[ V = \pi r^2 h \]
\[ V = \pi (2)^2 (5) \]
\[ V = 62.8 \text{ in}^3 \]

2. Students in one mathematics class noticed that a local movie theater sold popcorn in different shapes of containers, for different prices. They wondered which of the choices was the best buy. Analyze the three popcorn containers below. Which is the best buy? Explain.

![Image of three popcorn containers with dimensions and prices]

\[ V = L \cdot W \cdot H \]
\[ V = (14) (10) (19) \]
\[ V = 2520 \text{ cm}^3 \]

\[ V = \pi r^2 h \]
\[ V = \pi (7)^2 (18) \]
\[ V = 2770.9 \text{ cm}^3 \]

\[ V = \frac{1}{3} \pi r^2 h \]
\[ V = \frac{1}{3} \pi (9)^2 (25) \]
\[ V = 2126.6 \text{ cm}^3 \]

\[ \frac{60.5 \text{ cm}^3}{$1} \]

Better Buy
3. A sphere has a volume of $7776\pi$ cubic inches. Find the surface area of the sphere.

$$V = \frac{4}{3}\pi r^3$$

$$7776 \pi = \frac{4}{3}\pi r^3$$

$$5882 = r^3$$

$$r = 18 \text{ in}$$

$$SA = 4\pi r^2$$

$$SA = 4\pi (18)^2$$

$$SA = 1296\pi \text{ or } 4071.5 \text{ in}^2$$

4. A right circular cylinder has an altitude of 11 feet and a radius of 5 feet. What is the lateral area, in square feet, of the cylinder, to the nearest tenth?

$$LA = 2\pi rh$$

$$LA = 2\pi (5)(11)$$

$$LA = 345.6 \text{ ft}^2$$

5. A right circular cylinder has a volume of 1,000 cubic inches and a height of 8 inches. What is the radius of the cylinder to the nearest tenth of an inch?

$$V = \pi r^2 h$$

$$1000 = \pi r^2 (8)$$

$$39.79 = r^2$$

$$r = 6.3 \text{ in}$$

6. Find the difference in the volumes of the cones created by rotating the triangle shown below around the x-axis and around the y-axis. Write your answer in terms of $\pi$.

\begin{align*}
\text{X-axis:} & \quad \text{H} = 3 \\
& \quad r = 2 \\
& \quad V = \frac{1}{3}\pi r^2 h \\
& \quad V = \frac{1}{3}\pi (2)^2 (3) \\
& \quad V = 4\pi
\end{align*}

\begin{align*}
\text{y-axis:} & \quad \text{H} = 2 \\
& \quad r = 3 \\
& \quad V = \frac{1}{3}\pi r^2 h \\
& \quad V = \frac{1}{3}\pi (3)^2 2 \\
& \quad V = 6\pi
\end{align*}

\text{Difference} = 2\pi