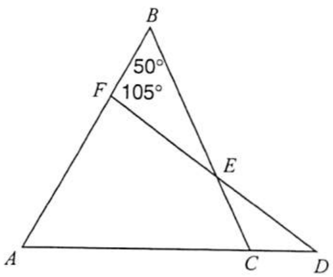
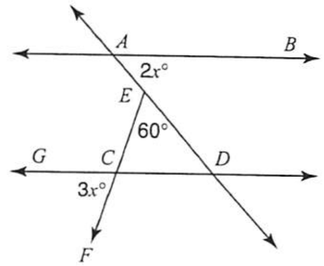
10. In the accompanying diagram of $\triangle ABC$, $\overline{AC}$ is extended to D, $\overline{DEF}$ is drawn, $m\angle B = 50$, $m\angle BFE = 105$, and $m\angle ACB = 65$. What is the measure of $\angle D$?
(1) 40 (2) 45 (3) 50 (4) 55



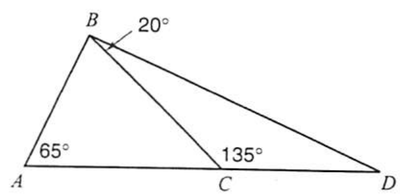
Exercise 10



Exercise 11

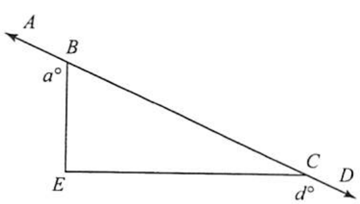
11. In the accompanying diagram, $\overleftrightarrow{AB} \parallel \overleftrightarrow{GCD}$, $\overleftrightarrow{AED}$ is a transversal, and $\overline{EC}$ is extended to F. If $m\angle CED = 60$, $m\angle DAB = 2x$, and $m\angle FCG = 3x$, what is the $m\angle GCE$?
(1) 36 (2) 72 (3) 108 (4) 144

12. In the accompanying diagram of $\triangle ABD$, C is a point on $\overline{AD}$, and $\overline{BC}$ is drawn.
Which statement must be true?
(1) $\overline{BC} \perp \overline{AD}$
(2) $\overline{AC} \cong \overline{CD}$
(3) $\overline{AB} \cong \overline{BD}$
(4) $\overline{AB} \perp \overline{BD}$

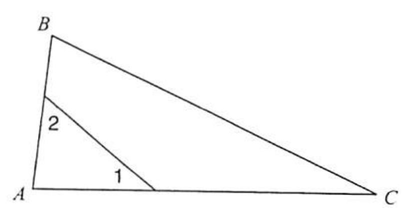


B. Show or explain how you arrived at your answer.

13. In the accompanying diagram, $\overleftrightarrow{ABCD}$ is a straight line, and $\angle E$ in $\triangle BEC$ is a right angle. What does $a + d$ equal?



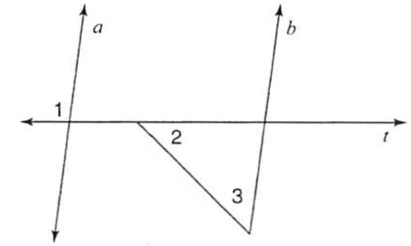
Exercise 13



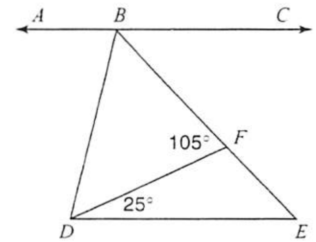
Exercise 14

14. In the accompanying diagram of $\triangle ABC$, $m\angle 1 = 40$, $m\angle 2 = 55$ and $m\angle B = 70$. Find $m\angle C$.

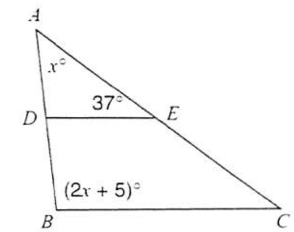
15. In the accompanying diagram, line a is parallel to line b, and line t is a transversal. If $m\angle 1 = 97$ and $m\angle 2 = 44$, find $m\angle 3$.



16. In the accompanying diagram, $\overleftrightarrow{ABC} \parallel \overline{DE}$, $\overline{BC}$ bisects $\angle DBC$, $m\angle FDE = 25$, and $m\angle DFB = 105$. What is $m\angle ABD$?



Exercise 16



Exercise 17

17. In the accompanying diagram, $\overline{DE} \parallel \overline{BC}$, $m\angle AED = 37$, $m\angle A = x$, and $m\angle B = 2x + 5$. What is the measure of $\angle ADE$?

2.4 POLYGONS AND THEIR ANGLES

KEY IDEAS

A triangle is the simplest type of *polygon.* It has three sides and three angles. A polygon with n sides has n interior angles. A simple formula can be used to find the sum of the measures of the interior angles of a polygon with any number of sides.

Definition of a Polygon

The term *polygon* means "many angles." A **polygon** is a closed figure whose sides are line segments that intersect at their endpoints. The polygon in Figure 2.9 has five sides. Each "corner" point where two sides intersect is a **vertex** of the polygon. The vertices of this polygon are A, B, C, D, and E. Line segment BE is a *diagonal.* A **diagonal** of a polygon is a line segment joining two nonconsecutive vertices.

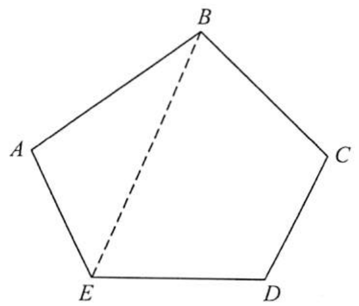


Figure 2.9 Polygon $ABCDE$ with diagonal $\overline{BE}$.

Sides	Name
4	Quadrilateral
5	Pentagon
6	Hexagon
8	Octagon
9	Nonagon
10	Decagon
12	Dodecagon
n	n–gon

Table 2.1
Classifying polygons

Beginning with any of its lettered vertices, a polygon is named by listing consecutive vertices in either clockwise or counterclockwise order. The polygon in Figure 2.9 can be named in many different ways including $ABCDE$ and $EDCBA$.

A **triangle** is a polygon with three sides. Table 2.1 lists other polygons and their names.

Convex Polygons

Polygon $ABCDEF$ in Figure 2.10 is **convex** since a line drawn through any two interior points intersects the polygon in exactly two points. The measure of each interior angle of a convex polygon is always less than 180.

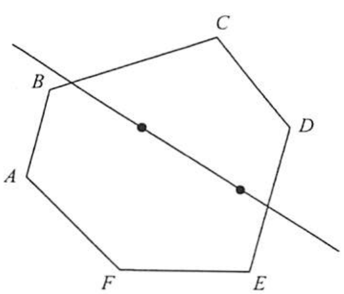


Figure 2.10 A convex polygon.

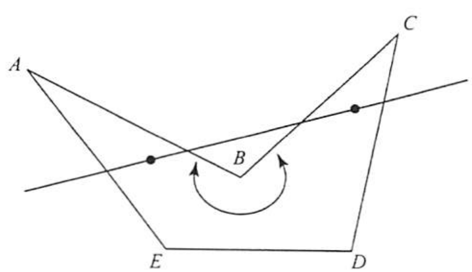


Figure 2.11 A concave polygon.

A polygon is nonconvex or **concave** if there are two points in the interior of the polygon such that the line through them (extended, if necessary) intersects the polygon in more than two points, as in Figure 2.11. The measure of the interior angle at vertex B is *greater than* 180, which also indicates that the polygon is concave. Only convex polygons are studied in this course.

Regular Polygons

In a polygon, the measures of the sides, the angles, or both the angles and sides may be equal. A **regular polygon** is a polygon in which each interior angle has the same measure (**equiangular**) *and* each side has the same length (**equilateral**). A square is a regular polygon because its four sides have the same length and its four right angles have the same measure.

Angle–Sum Formula for Polygons

A diagonal of a quadrilateral divides the quadrilateral into two triangles. The sum of the measures of the angles of these two triangles must be the same as the sum of the measures of the four angles of the quadrilateral. That sum is 2 × 180 or 360. Figure 2.12 shows that the number of triangles into which a polygon can be separated is always 2 less than the number of sides of the polygon.

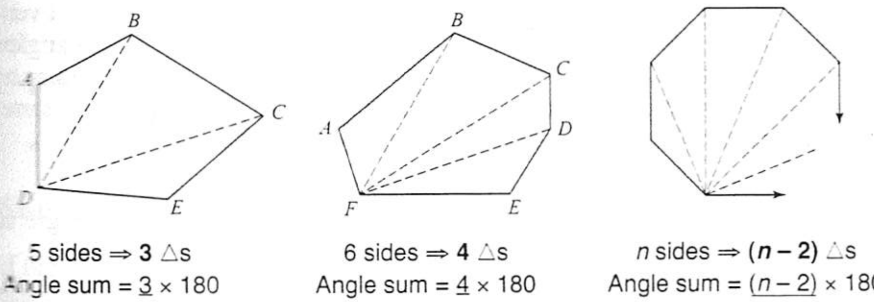


Figure 2.12 Dividing a polygon with n sides into $n - 2$ triangles.

> **Theorem: Polygon Angle–Sum Formula**
> The sum, S, of the measures of the interior angles of a convex polygon with n sides is given by the formula
> $$S = (n - 2) \times 180.$$

Example 1

What is the sum of the measures of the interior angles of a stop sign, which is in the shape of an octagon?

Solution: An octagon has eight sides. Evaluate the formula $S = (n - 2) \times 180$ for $n = 8$:

$$\begin{aligned} S &= (8 - 2) \times 180 \\ &= 6 \times 180 \\ &= \mathbf{1080} \end{aligned}$$

Example 2

Determine the number of sides of a polygon in which the sum of the measures of the angles is 900.

Solution: If n represents the number of sides of the polygon, then

$$900 = (n-2) \times 180$$
$$900 = 180n - 360$$
$$1260 = 180n$$
$$\frac{180n}{180} = \frac{1260}{180}$$
$$n = \mathbf{7\ sides}$$

Exterior Angles of a Polygon

At each vertex of a polygon, an *exterior* angle may be formed by extending one side of the polygon such that the interior and exterior angles at that vertex are supplementary. In Figure 2.13, angles 1, 2, 3, and 4 are exterior angles and the sum of their degree measures is 360. If the polygon is equiangular or regular, then each exterior angle, as well as each interior angle, has the same measure. For a regular pentagon,

- The sum of the measures of the five exterior angles is 360.
- As the polygon is equiangular, the measure of each exterior angle is $\frac{360}{5} = 72$.
- Since an exterior angle is supplementary to an interior angle at each of the vertices, the measure of each interior angle is $180 - 72 = 108$.

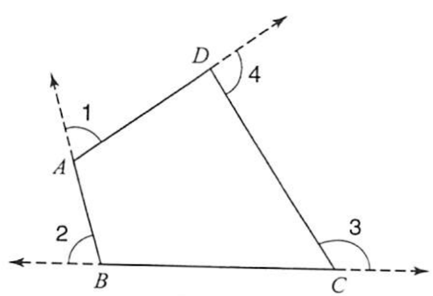


Figure 2.13 Exterior angles of a polygon.

Exterior Angle Theorems for Polygons

- **Theorem: Sum of Exterior Angles**
  The sum of the measures of the exterior angles of a convex polygon with any number of sides, one exterior angle at each vertex, is 360.
- **Theorem: Interior and Exterior Angle Relationships**
  If a *regular* polygon has n sides, then the measure of each exterior angle is $\frac{360}{n}$ and the measure of each interior angle is $180 - \left(\frac{360}{n}\right)$.

Example 3

Find the measure of each interior angle and each exterior angle of a regular decagon.

Solution: A decagon has ten sides.

- The measure of each exterior angle is $\frac{360}{10} = \mathbf{36}$.
- The measure of each interior angle is $180 - 36 = \mathbf{144}$.

Example 4

If the measure of each interior angle of a regular polygon is 150, how many sides does the polygon have?

Solution: Let n represent the number of sides of the regular polygon.

- Since each interior angle measures 150, each exterior angle measures
  $$180 - 150 = 30.$$
- As $\frac{360}{n} = 30$, $30n = 360$ so $n = \frac{360}{30} = \mathbf{12\ sides}$.

Check Your Understanding of Section 2.4

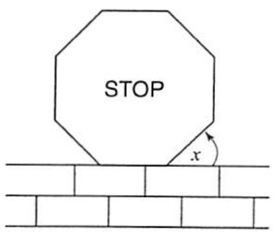
A. Multiple Choice

1. The number of sides of a regular polygon for which the measure of an interior angle is equal to the measure of an exterior angle is
(1) 8 (2) 6 (3) 3 (4) 4

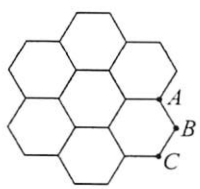
2. If the measure of an interior angle of a regular polygon is 108, what is the total number of sides in the polygon?
(1) 5 (2) 6 (3) 7 (4) 4

3. If the measures of four interior angles of a pentagon are 116, 138, 94, and 88, what is the measure of the remaining interior angle?
(1) 76 (2) 104 (3) 120 (4) 144

4. A stop sign in the shape of a regular octagon is resting on a brick wall, as shown in the accompanying diagram. What is the measure of angle x?
(1) 45 (2) 60 (3) 120 (4) 135



Exercise 4



Exercise 5

5. The accompanying figure represents a section of bathroom floor tiles shaped like regular hexagons. What is the measure of ABC ?
(1) 60 (2) 90 (3) 120 (4) 150

6. If the sum of the measures of the interior angles of a polygon is 1620, how many sides does the polygon have?
(1) 9 (2) 10 (3) 11 (4) 12

7. Which of the following could *not* represent the measure of an exterior angle of a regular polygon?
(1) 72 (2) 15 (3) 27 (4) 45

8. If each interior angle of a regular polygon measures 170, what is the total number of sides in the polygon?
(1) 10 (2) 17 (3) 18 (4) 36

B. Show or explain how you arrived at your answer.

9. What is the sum of the measures of the interior angles of a polygon that has 13 sides?

10. Find the number of sides of a regular polygon in which the measure of an interior angle is three times the measure of an exterior angle.

11. Find the number of sides in a regular polygon in which the measure of an interior angle exceeds six times the measure of an exterior angle by 12.