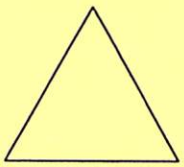
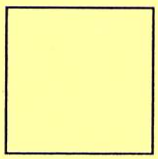
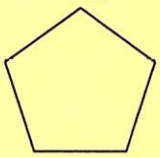
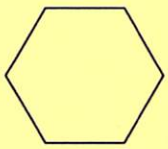
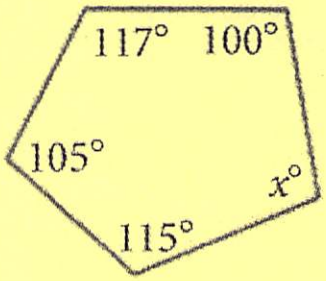
Interior and Exterior Angles in Polygons

Polygon	Number of Sides	Number of Triangles Formed	Sum of Interior Angles	**Regular Polygon** Single Interior Angle Measure	**Regular Polygon** Single Exterior Angle Measure	Sum of Exterior Angles
Triangle 	3	1	180°	60°	120°	360°
Quadrilateral 	4	2	360°	90°	90°	360°
Pentagon 	5	3	540°	108°	72°	360°
Hexagon 	6	4	720°	120°	60°	360°
Heptagon	7	5	900°	128.6	51.4°	360°
Octagon	8	6	1080°	135	45°	360°
Decagon	10	8	1440°	144°	36°	360°
Dodecagon	12	10	1800°	150°	30°	360°
15-gon	15	13	2340°	156°	24°	360°
60-gon	60	58	10440°	174°	6°	360°
***n*-gon**	n	$n-2$	$180(n-2)$	$\frac{180(n-2)}{n}$	$\frac{360}{n}$	360°

Some Practice:

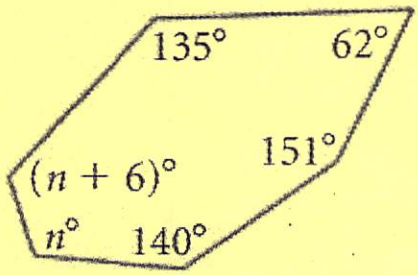
Find the missing angle measure.

1.



X = 103°

2.



n = 113°

3. Find the measure of an interior and exterior angle of a regular nonagon (9 sides).

Sum interior = 1260

∠ interior = 140°

∠ exterior 40°

4. The sum of the angles of a polygon is 1980°. Find the number of sides of the polygon.

180°

Δ's = 11

Sides = 13

13-gon

5. The measure of an exterior angle of a regular polygon is 18°. Find the measure of an interior angle *and find* the number of sides.

interior ∠ = 162°

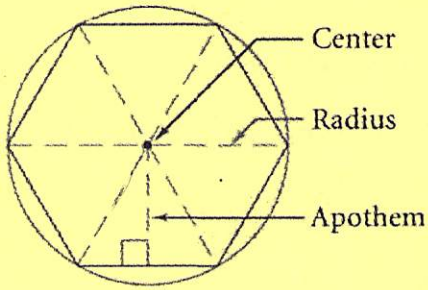
$$18^\circ = \frac{360}{n}$$

$$\frac{18n}{18} = \frac{360}{18}$$

n = 20 sides

Area of Regular Polygons

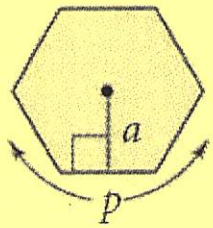
You can circumscribe a circle about any regular polygon. The **center** of the polygon is the center of the circumscribed circle. The **radius** is the distance from the center to a vertex. The **apothem** is the perpendicular distance from the center to a side.



To find the area of any regular *n*-gon you need the perimeter and the length of the apothem.

Area of a Regular Polygon

$A = \frac{1}{2} \cdot \text{apothem} \cdot \text{perimeter}$



*to find perimeter of an *n*-gon: n • side length

Examples:

1. Find the area of a regular decagon with a 12.3-in. apothem and 8-in. sides.

$A = \frac{1}{2} a \cdot p$

$a = 12.3$

$p = 80$

$A = \frac{1}{2}(12.3)(80)$

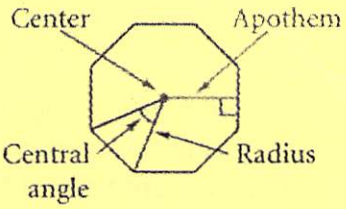
$A = 492 \text{ in}^2$

2. Find the area of a regular octagon with a 60.4-in. apothem and 50-in. sides.

$A = \frac{1}{2}(60.4)(400)$

$A = 12080 \text{ in}^2$

Trigonometry and Area



Example:

1. Find the area of a regular pentagon with 8-cm. sides.

Central $\angle = 72°$

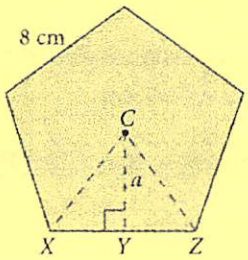
$\tan 36° = \frac{4}{a}$

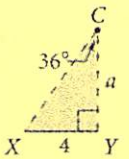
$a = 5.51$

$A = \frac{1}{2}ap$

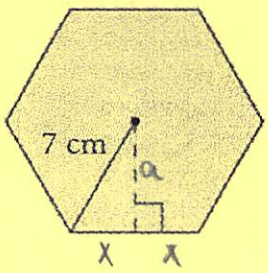
$A = \frac{1}{2}(5.51)(40)$

$A = 110.1\ cm^2$





2. Find the area of the following regular polygon *to the nearest tenth.*



Centra $\angle = 60°$

Side length $= 7$

$\sin 30 = \frac{x}{7}$

$x = 3.5$

$\cos 30 = \frac{a}{7}$

$a = 6.1$

$A = \frac{1}{2}ap$

$A = \frac{1}{2}(6.1)(42)$

$A = 127.3\ cm^2$

3. Find the area *to the nearest tenth* of a pentagon with side length 15 in.

Central $\angle = 72°$

$\tan 36 = \frac{7.5}{a}$

$a = 10.3$

$p = 75$

$A = \frac{1}{2}ap$

$A = \frac{1}{2}(10.3)(75)$

$A = 387.1\ in^2$