

# 10-3 Solving Quadratic Equations

Just like with linear equations, there are rules for solving **quadratic equations**. A quadratic equation is an equation that can be written in the form:

$$ax^2 + bx + c = 0$$

\* It turns out that the **solution(s)** to a quadratic equation is/are the **roots or zeros** of the quadratic function.

To solve quadratic equations, we will use a property that makes a lot of sense.

## Zero Product Property

For every real number  $a$  and  $b$ . If  $ab = 0$ , then  $a = 0$  or  $b = 0$ .

**Example:** If  $(x+3)(x+2) = 0$ , then  $x+3 = 0$  or  $x+2 = 0$

Let's use the Zero Product Property and solve for  $x$ .

$(x+7)(x-4) = 0$	$(3x-5)(4x-2) = 0$
$x+7=0$ -7 -7 $x=-7$	$x-4=0$ +4 +4 $x=4$

  

$\frac{3x-5}{+5+5} = \frac{5}{5}$	$\frac{4x-2}{+2+2} = \frac{2}{2}$
$x=\frac{5}{3}$	$x=\frac{2}{4} \text{ or } \frac{1}{2}$

Now we connect the Zero Product Property to solving quadratic equations.

Factor the Trinomial $x^2 - 2x - 15 = 0$ $\downarrow$ $(x-5)(x+3) = 0$ $x-5=0$ $+5 +5$ $x=5$	Difference of Perfect Squares $x^2 - 49 = 0$ $\downarrow$ $(x-7)(x+7) = 0$ $x-7=0$ $+7 +7$ $x=7$	GCF $x^2 + 4x = 0$ $\downarrow$ $x(x+4) = 0$ $x=0$
$\boxed{① = 0}$	$\boxed{x=-7}$	$\boxed{x=4}$

②  $x^2$  is positive

Examples:

1.  $x^2 + 11x + 18 = 0$

$$\begin{array}{c} 1, 18 \\ 2, 9 \\ 3, 6 \end{array}$$

$$(x+9)(x+2) = 0$$

$$\begin{array}{l|l} x+9=0 & x+2=0 \\ -9 -9 & -2 -2 \\ \hline x=-9 & x=-2 \end{array}$$

3.  $x^2 - 25 = 0$

$$(x+5)(x-5) = 0$$

$$\begin{array}{l|l} x+5=0 & x-5=0 \\ -5 -5 & +5 +5 \\ \hline x=-5 & x=5 \end{array}$$

5.  $2x^2 - 14x = 0$

$$2x(x-7) = 0$$

$$\begin{array}{l|l} 2x=0 & x-7=0 \\ \hline x=0 & +7 +7 \\ \hline x=0 & x=7 \end{array}$$

7.  $x^2 + 8x = -15$

$$\begin{array}{r} +15 +15 \\ \hline \end{array}$$

$$x^2 + 8x + 15 = 0$$

$$(x+3)(x+5) = 0$$

$$\begin{array}{l|l} x+3=0 & x+5=0 \\ -3 -3 & -5 -5 \\ \hline x=-3 & x=-5 \end{array}$$

7 concave up

2.  $2x^2 + 8x - 64 = 0$

$$\begin{array}{l|l} 2(x^2 + 4x - 32) = 0 & \\ \hline (x-4)(x+8) = 0 & \\ \hline 2 > 0 & x+8 = 0 \\ & +4 +4 -8 -8 \\ & x=4 x=-8 \end{array}$$

4.  $4x^2 - 4 = 0$

$$\begin{array}{l|l} 4(x^2 - 1) = 0 & \\ \hline 4(x-1)(x+1) = 0 & \\ \hline 4 > 0 & x-1=0 x+1=0 \\ & +1 +1 -1 -1 \\ & x=1 x=-1 \end{array}$$

Cubic Equation!!

6.  $x^3 + 7x^2 + 12x = 0$

$$\begin{array}{l|l} x(x^2 + 7x + 12) = 0 & \\ \hline x(x+3)(x+4) = 0 & \\ \hline x=0 & x+3=0 x+4=0 \\ & -3 -3 -4 -4 \\ & x=-3 x=-4 \end{array}$$

8.  $2x^2 - 8x - 12 = -2x + 8$

$$\begin{array}{r} -8 \\ \hline 2x^2 - 8x - 20 = -2x \\ +2x +2x \\ \hline 2x^2 - 6x - 20 = 0 \end{array}$$

$$2(x^2 - 3x - 10) = 0$$

$$2(x-5)(x+2) = 0$$

$$\begin{array}{l|l} x-5=0 & x+2=0 \\ +5 +5 -2 -2 \\ \hline x=5 x=-2 \end{array}$$

Solving Quadratic Equations Practice

Name \_\_\_\_\_

1.  $x^2 - x - 42 = 0$

2.  $3x^2 + 3x - 18 = 0$

3.  $x^2 - 81 = 0$

4.  $4x^2 - 64 = 0$

5.  $6x^2 + 12x = 0$

Cubic Equation!!

6.  $x^3 + 5x^2 + 14x = 0$

7.  $x^2 + 15x + 20 = 6x$

8.  $-4x^2 - 8x - 3 = -3 - 5x^2$